Mathematical Cell Biology Graduate Summer Course
University of British Columbia, May 1-31, 2012

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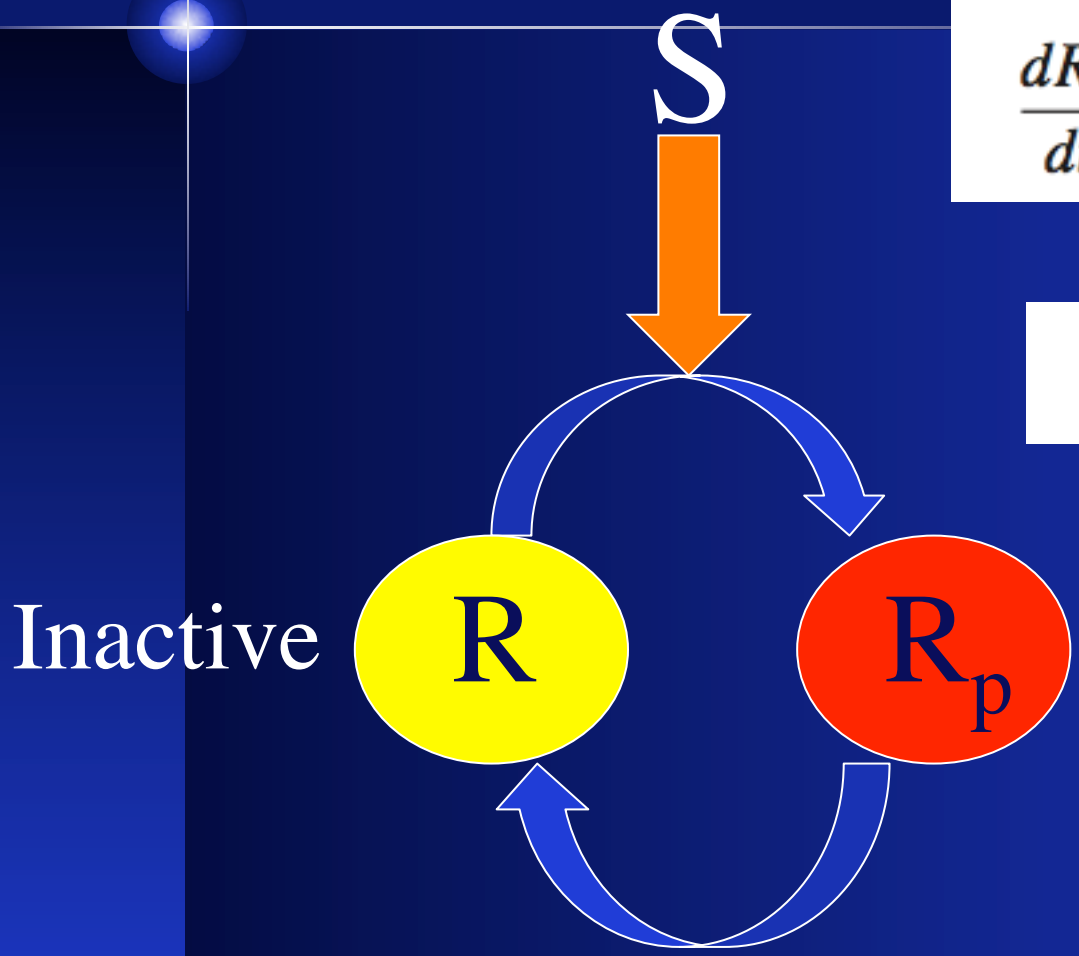
Simple biochemical motifs (3)

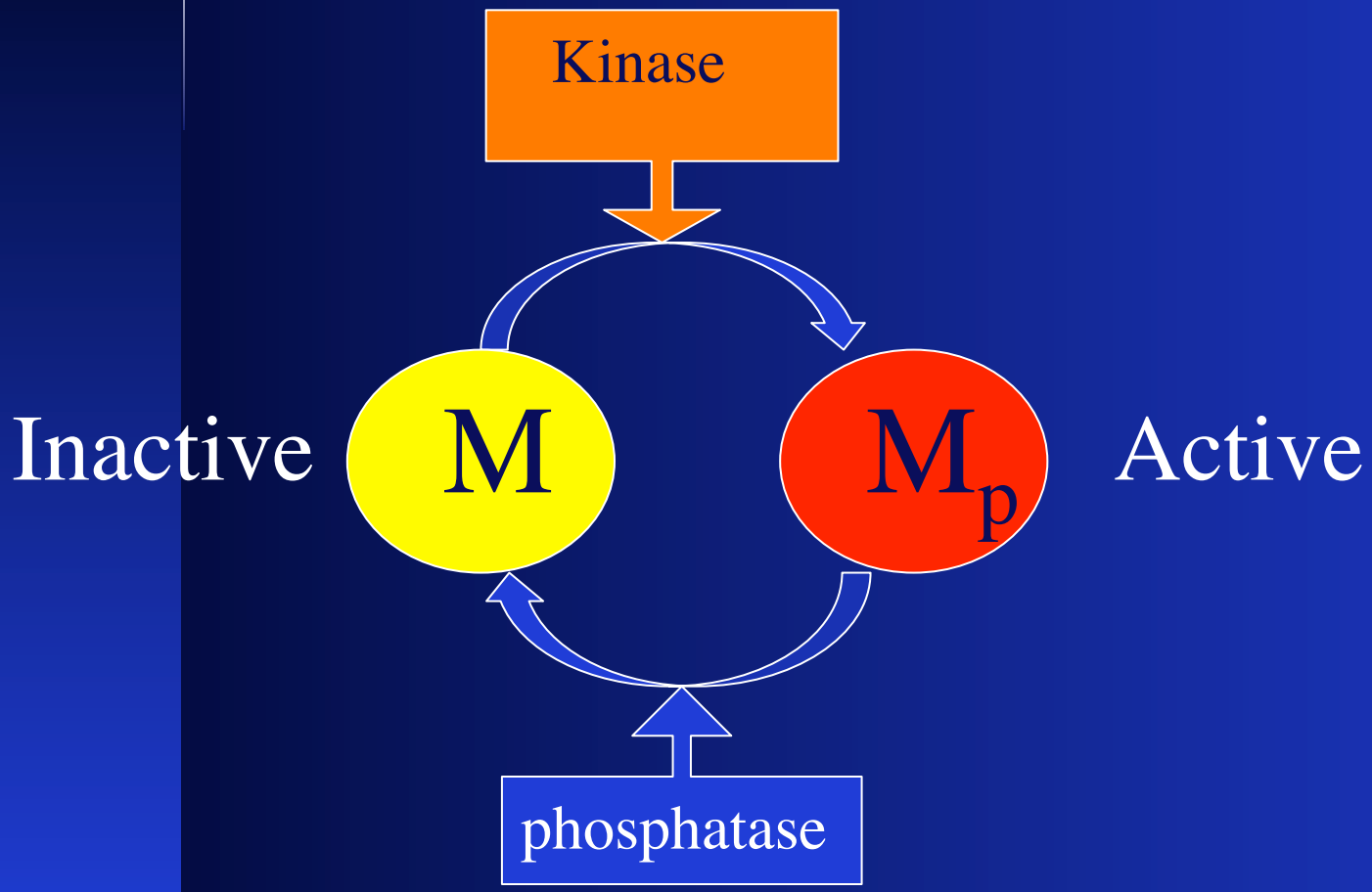
Activation-inactivation



$$\frac{dR_p}{dt} = \frac{k_1 SR}{K_{m1} + R} - \frac{k_2 R_p}{K_{m2} + R_p}.$$

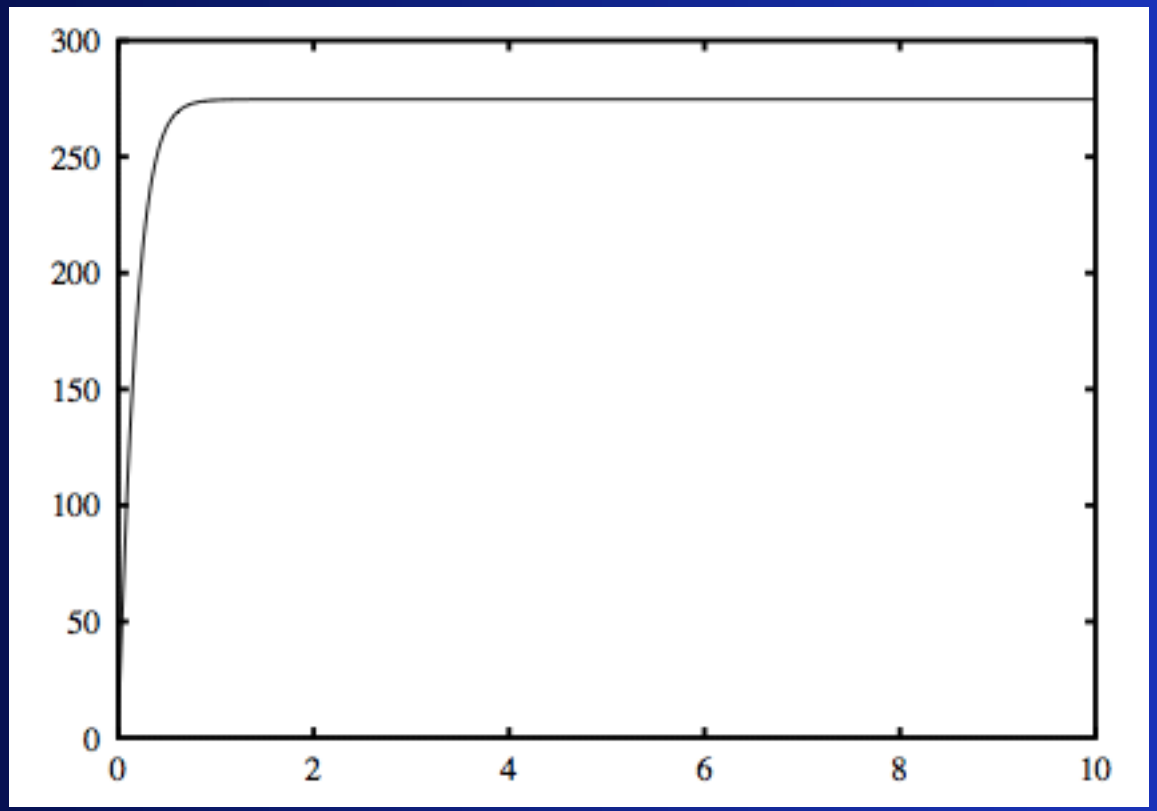
$$R_T = R + R_p = \text{ constant.}$$

GTPase cycle



Without feedback: Fast equilibration

M_p



Time (seconds)

System has a single biologically relevant steady state

Eliminate R, rescale

$$\frac{dR_p}{dt} = \frac{k_1 S R}{K_{m1} + R} - \frac{k_2 R_p}{K_{m2} + R_p}.$$

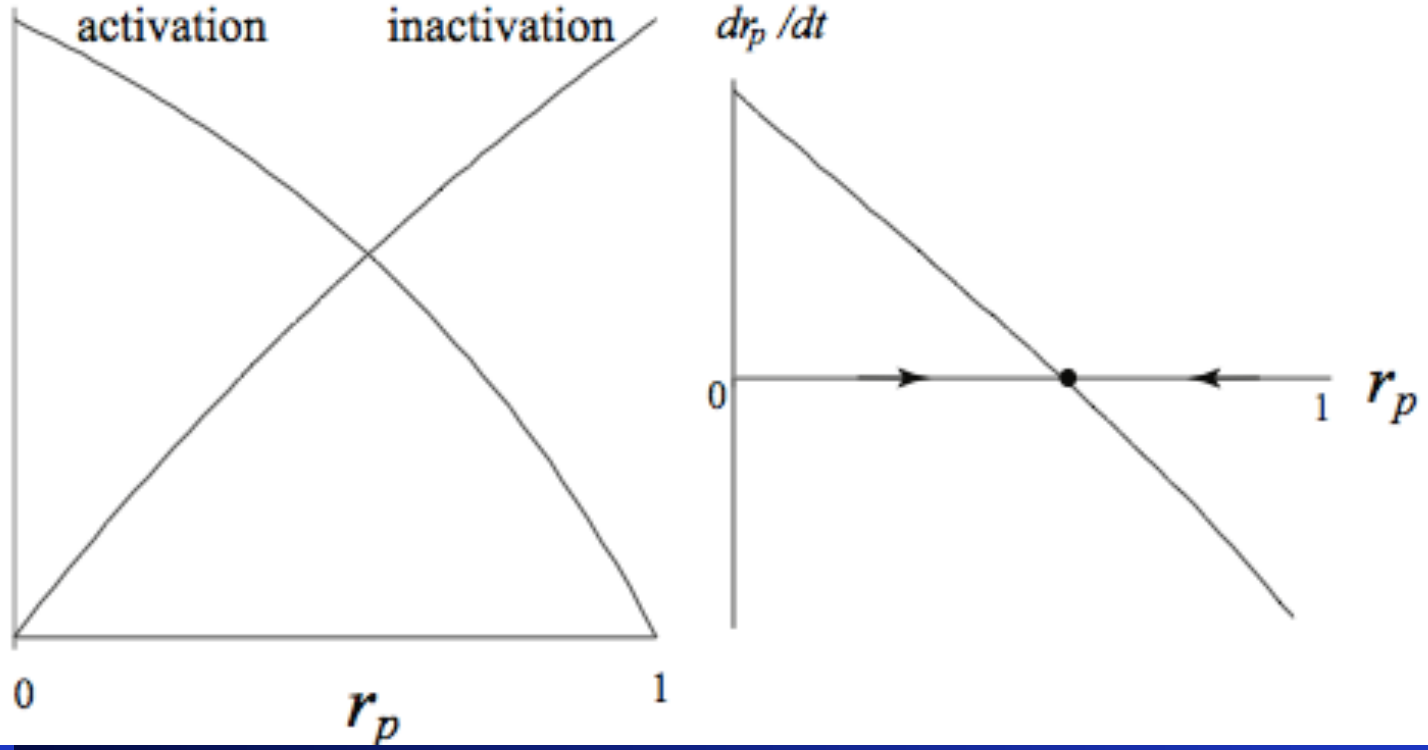
Use

$$R_T = R + R_p = \text{ constant.}$$

$$r_p = R_p / R_T$$

Rescaled

$$\frac{dr_p}{dt} = \frac{k_1 S(1-r_p)}{K'_{m1} + (1-r_p)} - \frac{k_2 r_p}{K'_{m2} + r_p}.$$



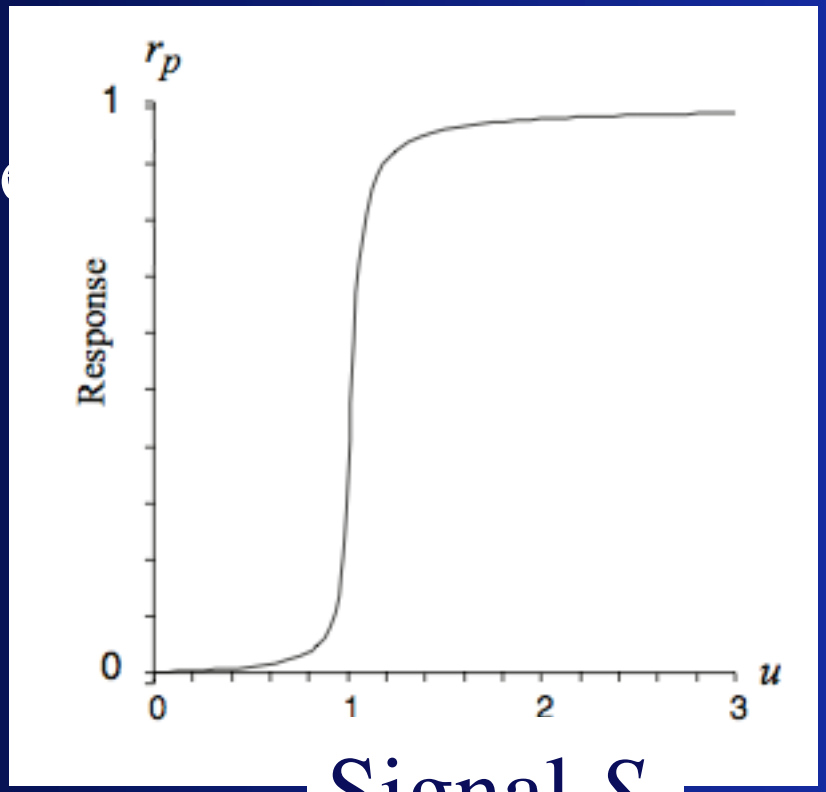
Steady states

$$\frac{dr_p}{dt} = \frac{k_1 S(1-r_p)}{K'_{m1}+(1-r_p)} - \frac{k_2 r_p}{K'_{m2}+r_p} = 0$$

The steady states can be shown to be solutions to a quadratic equation. Only one is positive and is called the "Goldbeter-Koshland function" of the stimulus.

"Zero order ultrasensitivity"

Steady state response



Signal *S*

response is minimal for low signal level, until some threshold. Then there is steep rise to full response. – *Goldbeter and Koshland*