Math 265 Nov 15, 2010

Systems of 1st order Linear ODES. -Review

Recall, we are studying

$$\left.\begin{aligned} \frac{dx}{dt} &= a_{11}x + a_{12}y \\ \frac{dy}{dt} &= a_{21}x + a_{22}y \end{aligned}\right\} \Rightarrow \frac{d\vec{x}}{dt} = M\vec{x} \qquad \vec{x} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \qquad \vec{M} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

We defined $\beta = Tr(M) = a_{11} + a_{22}$

$\gamma = \det(M) = a_{11}a_{22} - a_{21}a_{12}$

We showed that a soln to this system is (usually) of the form

$$\frac{d\vec{x}}{dt} = c_1\vec{V}_1 e^{r_1 t} + c_2\vec{V}_2 e^{r_2 t}$$

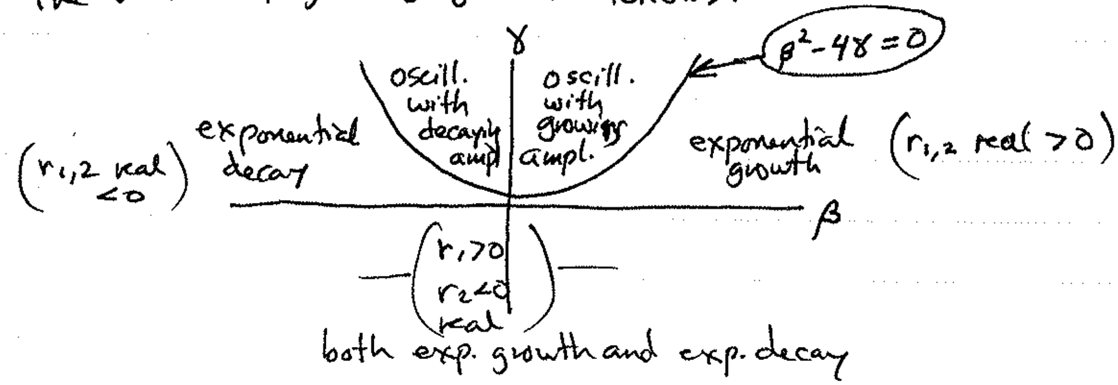
where $r_{1,2}$ are eigenvalues of M (satisfy $\det(M - rI) = 0$)

i.e. $$r_{1,2} = \frac{+\beta \pm \sqrt{\beta^2 - 4\gamma}}{2}$$

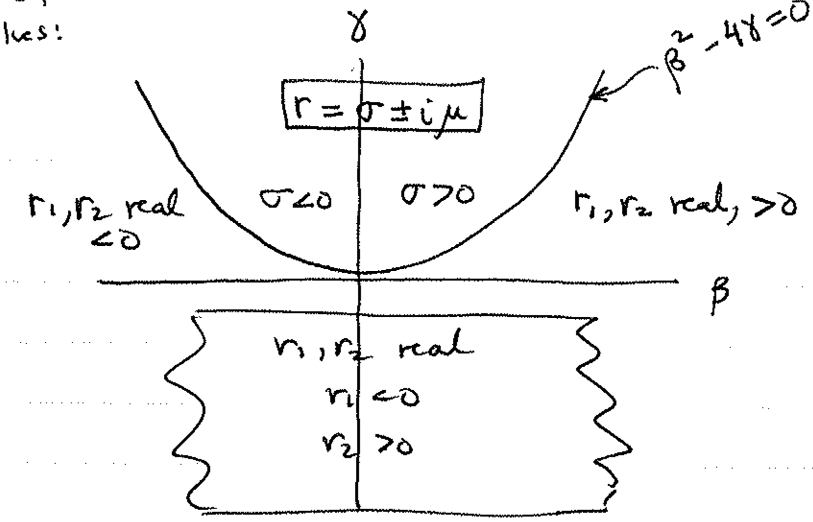
and $\vec{V}_{1,2}$ are eigenvectors of M (satisfy $(M - rI)\cdot\vec{v} = 0$)

i.e. $$\vec{V}_i = \begin{pmatrix} \frac{r_i - a_{22}}{a_{21}} \\ 1 \end{pmatrix}$$ ← (one of several possible forms for eigenvector)

We also classified the behaviour of the system according to the values of β and γ as follows:

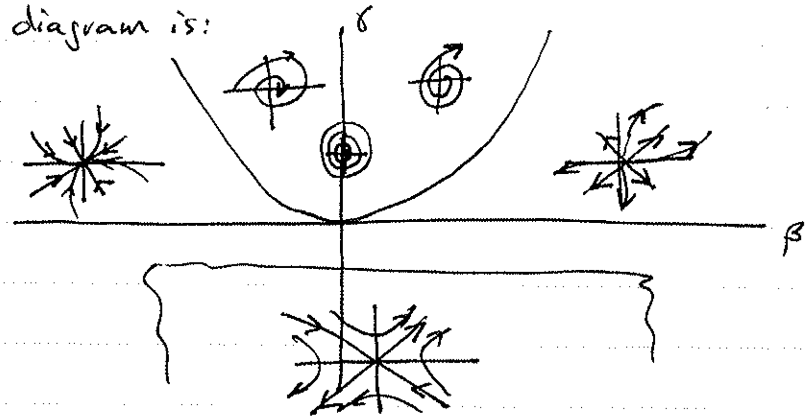


In terms of the types of eigenvalues:



To come:

We will also see that in terms of phase-plane behaviour, same summary diagram is:



Alternate forms of eigenvectors

$$\begin{pmatrix} a_{11}-r & a_{12} \\ a_{21} & a_{22}-r \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

① $(a_{11}-r)\,v_1 + a_{12}v_2 = 0$

② $a_{21}v_1 + (a_{22}-r)\,v_2 = 0$

Recall: these eqns are <u>NOT</u> linearly indep., so can use any <u>ONE</u> of them

e.g.

① $\Rightarrow$ set $v_1 = 1$ then $v_2 = \dfrac{r-a_{11}}{a_{12}}$

get the form $\begin{pmatrix} 1 \\ \frac{r-a_{11}}{a_{12}} \end{pmatrix}$ (we could also set $v_2 = 1$ and find v_1)

or $\begin{pmatrix} a_{12} \\ r-a_{11} \end{pmatrix}$

<u>ALTERNATELY</u>

② $\Rightarrow$ set $v_2 = 1$ then $v_1 = \dfrac{r-a_{22}}{a_{21}}$

get the form $\begin{pmatrix} \frac{r-a_{22}}{a_{21}} \\ 1 \end{pmatrix}$ or $\begin{pmatrix} r-a_{22} \\ a_{21} \end{pmatrix}$