Classify the following systems:

① $\begin{cases} \dfrac{dx}{dt} = x - 2y \\[2mm] \dfrac{dy}{dt} = 5x + y \end{cases}$

$M = \begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix}$

$\beta = \text{Tr } M = 2$

$\gamma = \det M = 1 + 10 = 11$

$\beta^2 < 4\gamma$ $\beta > 0$

$\Downarrow$ $\Downarrow$

(oscillations)   (growing)

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② $\begin{cases} \dfrac{dx}{dt} = -x + y \\[2mm] \dfrac{dy}{dt} = x - 3y \end{cases}$

$M = \begin{bmatrix} -1 & 1 \\ 1 & -3 \end{bmatrix}$

$\beta = \text{Tr } M = -4$

$\gamma = \det M = 3 - 1 = 2$

$\beta^2 = 16$ $4\gamma = 8$

$\beta^2 > 4\gamma$ $\beta < 0$

$\Downarrow$ $\Downarrow$

exponential   (decaying)
solns

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③ $\dfrac{dx}{dt} = -2x + 5y$

$\dfrac{dy}{dt} = x - y$

$M = \begin{bmatrix} -2 & 5 \\ 1 & -1 \end{bmatrix}$

$\beta = -3$

$\gamma = 2 - 5 = -3 < 0$

$\left(\beta^2 - 4\gamma = 9 + 12 = 21 > 0 \right) \leftarrow$ (not needed)

$\gamma < 0 \Rightarrow$ real roots of opposite
signs

exponentials with 1 growing and
1 decaying.

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④ $\dfrac{dx}{dt} = y$

$\dfrac{dy}{dt} = -3x$

$M = \begin{bmatrix} 0 & 1 \\ -3 & 0 \end{bmatrix}$

$\beta = 0$

$\gamma = 3$

$\beta^2 - 4\gamma = -12 < 0 \Rightarrow$ oscillations

$\beta = 0 \Rightarrow$ osc. do not grow nor decay
constant ampl.