Sept 27

How do we interpret $e^{iwt}$ when $i = \sqrt{-1}$ ?

Taylor Series:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

let $x = it$ where $i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$ etc.

$$e^{it} = 1 + (it) + \frac{(it)^2}{2!} + \dots$$

$$= 1 + it - \frac{t^2}{2!} - \frac{it^3}{3!} + \frac{t^4}{4!} + \dots$$

$$= \underbrace{\left(1 - \frac{t^2}{2!} + \frac{t^4}{4!}\right)}_{\text{real}} + i \underbrace{\left(t - \frac{t^3}{3!} + \frac{t^5}{5!} + \dots\right)}_{\text{imag. part}}$$

$$= \cos(t) + i \sin t$$

$$\boxed{e^{it} \equiv \cos(t) + i \sin(t)}$$

or let $x = iwt$:

$$\boxed{e^{iwt} = \cos(wt) + i \sin(wt)}$$