

Useful Trig identities:

$$I1 \quad \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$I2 \quad \sin(A-B) = \sin A \cos B - \sin B \cos A$$

$$I3 \quad \sin^2 \theta + \cos^2 \theta = 1$$

$$I1 \Rightarrow \cos(\omega t - \delta) = \cos(\omega t) \cos \delta + \sin(\omega t) \sin \delta$$

$$I2 \Rightarrow \sin(\omega t - \delta) = -\cos(\omega t) \sin \delta + \sin(\omega t) \cos \delta$$

} → if we eliminate  $\sin(\omega t)$  from these two eqns, we get  
←

$$\cos \delta \cos(\omega t - \delta) - \sin \delta \sin(\omega t - \delta) = \cos \omega t$$

Other derived formulae

For Beats

$$(a) \quad \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$(b) \quad \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$(a)-(b): \quad \cos(A-B) - \cos(A+B) = 2 \sin A \sin B$$

I4

$$\cos(u) - \cos(v) = 2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{v-u}{2}\right)$$

now let  $u = A-B$   
 $v = A+B$

then  $A = \frac{u+v}{2}$     $B = \frac{v-u}{2}$

Also by I1:

$$R \cos(\omega t - \delta) = [R \cos \delta] \cos(\omega t) + [R \sin \delta] \sin(\omega t)$$
$$= A \cos(\omega t) + B \sin(\omega t)$$

$$\Rightarrow \begin{cases} A = R \cos \delta \\ B = R \sin \delta \end{cases}$$

$$A^2 + B^2 = R^2 \cos^2 \delta + R^2 \sin^2 \delta = R^2$$

$$\Rightarrow \begin{cases} R = \sqrt{A^2 + B^2} \\ \tan \delta = \frac{B}{A} \end{cases}$$