Sample Final Examination

Mathematics 104/184

All Sections

[42] 1. **Short Problems.** Each question is worth 3 points. Put your answer in the box provided and show your work. No credit will be given for the answer without the correct accompanying work.

(a) Evaluate $\lim_{x \to -3} \dfrac{2x^2 + 5x - 3}{x^2 + x - 6}$.

Answer:

(b) Suppose that the demand curve for a certain commodity is modeled by $p = \dfrac{3400q + 1}{2q^2 + q}$, where p is the price and q is the quantity. Find the limiting value of the revenue R as $q \to \infty$.

Answer:

(c) How much money must initially be put in an account paying 8% annual interest, compounded continuously, in order to have \$3000 at the end of 30 months? You may leave your answer in calculator-ready form.

Answer:

(d) Find the slope of the graph of $y = f\big(g(x^2)\big)$ at $x = 1$, where $f'(0) = 3$, $g(1) = 0$ and $g'(1) = -1$.

Answer:

(e) Find the x-coordinates of all points where the tangent line to the graph of $y = \tan^{-1}(3x)$ is parallel to the line $3x - 2y + 7 = 0$. Note that $\tan^{-1}$ refers to the inverse tangent function, which is also denoted by arctan.

Answer:

(f) Suppose the weight of an animal at time t is given by $\omega = \dfrac{216}{\sqrt{t+5}} + t^{3/2}$ for $t \geq 0$. Use a derivative to determine whether the animal is gaining or losing weight when $t = 4$.

Answer:

(g) A company produces 500 radios per day. At this production level, the marginal daily revenue is $2.50 per radio. Estimate the change in the daily revenue if the company increases production by 20 radios per day.

Answer:

(h) If a function $y = f(x)$ is differentiable at $x = 2$ and $f'(2) = 6$, find the limit $\lim_{h \to 0} \dfrac{3h}{f(2) - f(2+h)}$.

Answer:

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(i) If the derivative of $f(x)$ is given by $f'(x) = \dfrac{2x}{1+x^2}$, find the interval or intervals on which $f(x)$ is concave up.

Answer:

(j) Find the values of the constants a and b such that the function $f(x) = x^3 + ax^2 + bx + c$ has critical points at $x = 0$ and $x = 1$.

Answer:

(k) Find the point in the first quadrant $(x > 0, y > 0)$ on the hyperbola $x^2 - y^2 = 4$ which is closest to the point $(6, 0)$. You do not need to justify that your answer provides the minimum.

Answer:

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(l) Suppose the demand function for a certain product is given by $q = 2000e^{-kp}$, where $k > 0$ is a constant, p is the unit price and q is the quantity sold. Determine the value of k if the maximum revenue occurs at $p = 200$.

Answer:

(m) Let $c_0 + c_1x + c_2x^2 + c_3x^3$ be the third order Taylor polynomial for the function $f(x) = \ln(2 + \cos x)$ with its center at 0. Determine the value of c_2.

Answer:

(n) Evaluate $\lim_{x\to\infty} \left(\frac{3x^5 - 2x + 7}{2x^5 - 3} \left(\frac{1}{e^x} + 5 \right) \right)$.

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Long Problems. In questions 2–6, show your work. No credit will be given for the answer without the correct accompanying work.

[10] **2.** Let $f(x) = \dfrac{1}{\sqrt{3-x}}$ for $x < 3$. Use the definition of the derivative to find $\dfrac{df}{dx}$ for $x < 3$. No marks will be given for the use of any differentiation rules.

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[14] **3.** Let $y = f(x)$ be a function defined for $-\infty < x < \infty$ such that

$$f(0) = 1, \lim_{x \to \infty} f(x) = 1, f'(x) = (2x - x^2)e^{-x}.$$

(a) Find the interval or intervals on which $f(x)$ is decreasing.

(b) Find the interval or intervals on which $f(x)$ is concave down.

(c) Sketch the graph of $y = f(x)$ indicating where all local maxima, local minima and inflection points occur. (Hint: $\sqrt{2} \approx 1.4$.)

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[15] **4.** A right cylindrical can is to be constructed to hold 375π m^3 of oil. The cost of the material used for the top and bottom of the can is \$3/m^2 and the cost of the material used for the curved side is \$2/m^2. Find the radius of the can that will minimize the cost of the material used to construct the can. You do not need to justify that your answer provides the minimum.

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[12] **5.** A point is moving along the x-axis in the positive direction at a constant rate of 5 units per second.

(a) Where is the point when its distance from the point $(0, 1)$ in the xy-plane is increasing at a rate of 4 units per second?

(b) Where is the point when its distance from the point $(0, 1)$ in the xy-plane is increasing at a rate of 6 units per second? Explain.

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[7] **6.** At the present instant the consumer price index (CPI) in a country is 120, and is increasing at a rate of 10 points per year. Also, at the present instant this rate of increase is itself increasing at a rate of 4 points per year^2. Use calculus and **all** of this information to estimate the CPI in the country 6 months from now.

The End