More Solutions to Mid-Term I

3 (b) A function $g(x)$ equals $\sqrt{7+\frac{\sin 2x}{x}}$ for $x \neq 0$, but its value at $x = 0$ is kept secret.
How much must this value be if it is found out that $g(x)$ is actually a continuous function for $-\infty < x < \infty$?

From continuity at $x=0$ we have $\lim_{x\to 0} g(x) = g(0)$.
[See formula (1) in Text, P.88].
Hence the value of $g(x)$ at 0 can be computed via limits:

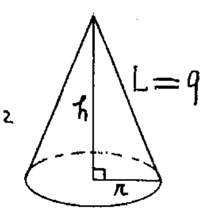
$$g(0) = \lim_{x\to 0}\sqrt{7+\frac{\sin 2x}{x}} = \lim_{x\to 0}\sqrt{7+2\cdot\frac{\sin 2x}{2x}} = \sqrt{7+2\cdot 1}$$

$$= \sqrt{9} = 3, \quad \text{using } \lim_{x\to 0}\frac{\sin 2x}{2x} = 1$$

4. A right circular cone of base radius r and height h has slant height $L = \sqrt{r^2+h^2}$.
What is the maximal possible volume of a cone with slant height equal to 9 cm ?

To be maximized is

$V = \frac{\pi}{3} r^2 h$ where r, h have relationship $r^2 + h^2 = 9^2$



Thus $V = \frac{\pi}{3}(81-h^2)h = \frac{\pi}{3}(81h - h^3)$, $0 \leq h \leq 9$.

Note: clearly, it is better to replace r^2 by $(81-h^2)$ than to replace h by $\sqrt{81-r^2}$, even though this latter step will also give a correct answer.
Note We must not make the bad error thinking $\sqrt{81-r^2}$ to be the same as $9-r$. In general $\sqrt{A^2 \pm B^2} \neq A \pm B$

Now $V' = \frac{\pi}{3}(81-3h^2)$
Setting $V'=0$ gives $3h^2 = 81$; $h=\sqrt{27}$ is critical point.
Since $V=0$ for $h=0,9$
V_{max} is realized at $h=\sqrt{27}$, $r=\sqrt{54}$, with
$V_{max} = 54\pi\sqrt{3}$ cm^3