Math 309: Introduction to knot theory

Homework 1, due Friday January 18 by 5:00 pm.

1. Recall, from class, that for a pair of relatively prime positive integers p and q we can form a knot with parametrization

$$x(t) = (2 + \cos(pt))\cos(qt)$$
$$y(t) = (2 + \cos(pt))\sin(qt)$$
$$z(t) = \sin(pt)$$

where $0 \leq t \leq 2\pi$.

(a) Find a knot diagram for the knot you obtain in the case $p = 1, q = 2$ by projecting to the xy-plane. Is this knot trivial? Why or why not?

(b) Find knot diagrams for the knot in the case $p = 3, q = 2$ and the knot in the case case $p = 2, q = 3$ by projecting to the xy-plane, and exhibit a sequence of Reidemeister moves establishing that these two knots are equivalent.

2. (Adams, Exercise 1.7) Show that by changing some of the crossings from over to under or vice versa, any projection of a knot can be made into a projection of the unknot.

3. (Adams, Exercises 1.18 and 1.19) Give the definition of a *Brunnian* link and:

(a) Find a Brunnian link of four components.

(b) Find a Brunnian link with arbitrarily many components.

4. (Adams, Exercise 1.21) There are three knots admitting minimal diagrams with 6 crossings (they are denoted 6_1, 6_2, and 6_3 and shown in the appendix of *the knot book*). Determine which of these six-crossing knots is tricolourable.

5. (Adams, Exercise 1.23) Show that the Type III Reidemeister move preserves tricolourability.

6. Prove that the trefoil knot (3_1) and the figure-eight knot (4_1) are distinct knots by appealing to tri-colourability.

7. (Adams, Exercise 1.28) Label the strands of the figure-eight knot with a selection of integers from the set $\{0, 1, 2, 3, 4\}$, using at least two integers, so that they satisfy $x + y - 2z = 0$ (modulo 5) at each crossing, where z labels the overstrand.

(a) Show that such a labeling system on a knot projection is preserved under the Reidemeister moves, and conclude that the figure eight knot is not the trivial knot.

(b) Reinterpret tricolouration in terms of a numerical scheme like the one used above.