Math 309: Introduction to knot theory

Homework 2, due Wednesday February 6 by 12:00 pm

(Submit your work at Room 219 of the Mathematics building)

1. (Adams, Exercise 2.14) Show that the rational tangle associated with $[2\ 1\ a_1\ a_2\ \ldots\ a_n]$ is equivalent to the rational tangle associated with $[-2\ 2\ a_1\ a_2\ \ldots\ a_n]$ by both using continued fractions and by comparing the relevant tangle diagrams.

2. Prove that 2-bridge knots are alternating in two steps:

(a) (Adams, Exercise 2.17) Prove that every rational knot is alternating (by finding an alternating diagram).

(b) Prove that every 2-bridge knot is a rational knot.

3. Show that the unknotting number of the rational knot associated with [5 1 4] is at least 2.

4. Give the definition of a *composite knot* and, by appealing to the fact that

$$b(K_1 \# K_2) = b(K_1) + b(K_2) - 1$$

(proved by Schubert), show that all rational knots are prime.

5. (Adams, Exercise 6.8) Let $V(L)$ be the Jones polynomial of a link, and prove that

$$t^{-1}V(L_+) - tV(L_-) = (t^{\frac{1}{2}} - t^{-\frac{1}{2}})V(L_0)$$

where each of the links L_+, L_-, and L_0 differ at exactly one crossing, as in Figure 6.13 of Adams.

6. Calculate the Jones polynomial of the figure 8 knot.