Math 309: Introduction to knot theory

Homework 3, due on Wednesday March 6 by 12:00 pm

(Submit your work at Room 219 of the Mathematics building)

1. Submit a complete solution to Problem 3 from the midterm.

2. Following the discussion in class, define the mutant of a knot K and prove that mutation preserves the bracket polynomial.

3. Prove that the quantity $\mathrm{Span}\big(\langle K\rangle\big)$ is a knot invariant, and check that $\mathrm{Span}\big(\langle K\rangle\big) = 4\,\mathrm{Span}\big(V_K(t)\big)$.

4. Complete the calculation of the $\langle K\rangle$ from class, where K is the (right hand) trefoil, using the formula

$$\langle D\rangle = \sum_S A^{a(S)-b(S)}(-A^{-2}-A^2)^{|S|-1}$$

(where D is a knot diagram).

5. Compute the Alexander polynomial of the figure eight knot, and show that the span of the Alexander polynomial is strictly smaller than the span of the Jones polynomial in this case.