Math 309: Introduction to knot theory

Additional questions for review, collected from the final lectures

(not for submission)

1. Show that every braid diagram β admits and inverse braid diagram β^{-1} so that, after isotopy, $\beta \cdot \beta^{-1}$ and $\beta^{-1} \cdot \beta$ (each obtained by concatenation) are equivalent to the trivial braid.

2. Show, via an explicit example, that multiplication in the braid group (on 3 or more strands) is not commutative. For 2-strand braids, multiplication is commutative. Can you explain why?

3. Find the Artin combed form for the braid $(\sigma_2\sigma_1^{-1})^3$.

4. Find all 14 so-called crossingless matchings of 8 points in the boundary of disk (you can think of these as two sets of 4 points on opposite sides of a square). Recall from class that these matchings are a basis for $\mathcal{TL}_4$.

5. Given a braid $\beta \in B_n$, write down a formula for the Jones polynomial $V_{\overline{\beta}}(t)$ in terms of the value $\mathcal{B}(\beta) \in \mathcal{TL}_n$. (Hint: try the case $n = 3$ first)

6. In the case of $\beta \in B_3$, determine the effect of an M2 move on $\mathcal{B}(\beta)$, that is, calculate $\mathcal{B}(\sigma_4^{\pm 1}\beta)$.