Math 427/527: algebraic topology

Homework 2, due Wednesday February 26 by 5:00 pm.

1. Let $f\colon S^n \to S^n$ be a map where $n > 0$ and for which $f^{-1}(y) = \{x_1, \ldots, x_m\}$.
(i) Calculate $H_n(S^n, S^n \smallsetminus f^{-1}(y))$.
(ii) Let k_i be the map induced by inclusion

$$(U_i, U_i \smallsetminus x_i) \to (S^n, S^n \smallsetminus f^{-1}(y))$$

(where U_i is a neighbourhood of x_i) and let p_j be the map induced by projection

$$(S^n, S^n \smallsetminus f^{-1}(y)) \to (S^n, S^n \smallsetminus x_j)$$

Show that $p_j \circ k_i$ vanishes when $i \neq j$.

2. Let X be a CW complex. Using the mapping telescope construction (Hatcher, page 138) complete the last step of the proof given in class on February 7: show that $\widetilde{H}_n(X^{(n+1)}) \cong \widetilde{H}_n(X)$.

3. Let $f\colon S^1 \to S^1$ be a map. Making explicit appeal to our definition of local degree, prove that $\deg(f)$ agrees with the winding number of f.

4. For each $d \in \mathbb{Z}$ and for each $n \in \mathbb{Z}_{>0}$ describe a surjective map $S^n \to S^n$ of degree d.

5. In this problem you will need to use homogeneous coordinates $(z_1 : z_2 : \cdots : z_n)$ in order to work with complex projective space $\mathbb{C}\mathrm{P}^{n-1}$. Where needed, you may assume tools from the theory of smooth manifolds; just be sure to record what you are making use of.
(i) Find an explicit CW structure for $\mathbb{C}\mathrm{P}^2$ and calculate the unreduced homology groups.
(ii) Similarly, find a CW structure for

$$F = \{(x : y : z) | x^2 + y^2 + z^2 = 0\} \subset \mathbb{C}\mathrm{P}^2$$

and calculate the unreduced homology groups.
(iii) The K3 surface is the subspace of $\mathbb{C}\mathrm{P}^3$ given by

$$X = \{(x : y : z : w) | x^4 + y^4 + z^4 + w^4 = 0\}$$

which is a (smooth) 4-manifold. Show that X has Euler characteristic 24.