Math 309: Introduction to knot theory

Assignment 2, due October 5 by 11:59 pm.

Exercises.

1. Prove that each of the 6-crossing knots in the knot table has unknotting number 1.

2. Find a family $\{\mathcal{L}_n : n \in \mathbb{Z}\}$ such that $\mathcal{L}_n$ is an oriented two component link whose linking number is n.

3. Define the "linking number of a knot" as follows: take a knot K, orient it, and assign $+1$ or -1 to each crossing, as in the definition of linking number; finally, take the sum of these $+1$s and -1s. Show that this procedure does not produce a knot invariant.

4. Based on material that we have seen so far in this course, show that the following links are non-trivial:

Problems.

5. Recall that McCoy's theorem states that an alternating knot with unknotting number 1 admits an unknotting crossing in any minimal alternating diagram. Consider the following diagram then:

For this problem, let this represent a knot referred to as K.

(a) Show that, based on the diagram given, the unknotting number of K is *at most* 2.

(b) Find an alternating diagram for K, and using this give a proof that the unknotting number of K is *exactly* 2.

6. A **simple link** is a 2-component link with the property that if it can be isotoped so that there are *exactly* two crossings between the link components.

(a) Give an example, with proof, of a simple link. And, give an example, with proof, of a 2-component link that is not simple.

(b) Consider a simple link L with the property that each component is a 3-colourable knot. Prove that the link L is 3-colourable.