Math 309: Introduction to knot theory

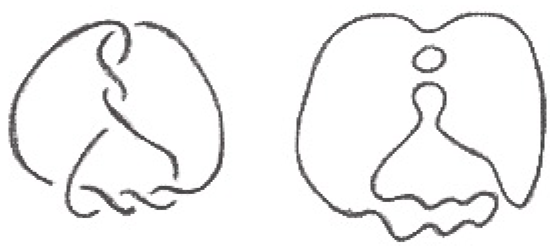
Assignment 5, due Monday November 9 by 11:59 pm.

Exercises.

1. Show that the span of the Jones polynomial of a reduced alternating diagram gives exactly the crossing number of the diagram.

2. Fix a reduced alternating diagram D. Let S_B be the state associated with D that is obtained by considering B-splits are every crossing. Show that the bottom power that this state contributes to the bracket polynomial is $-n - 2(\mathcal{S} - 1)$, where n is the number of crossings in D and $\mathcal{S}$ is the number of shaded regions in the checkerboard shading of D.

3. Consider the following diagram and the associated state S_1 shown:



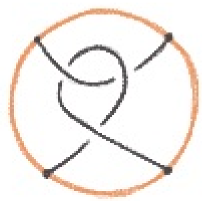
List all states S_2 that can be obtained from S_1, switching one resolution, so that $b(S_2) = b(S_1) + 1$.

4. During lectures, we proved that the bracket polynomial is unchanged by mutation. What conditions on a tangle need to be satisfied in order to ensure that the Jones polynomial is not changed by a mutation? What does this imply about the Jones polynomial of a knot and its mutants? Be sure to Justify your answers!

Problems

5. Let D be a reduced alternating diagram, and let S_A be the state obtained from D by choosing the A-split at every crossing. Show that the highest power of A contributed to $\langle D \rangle$ by S_A is strictly larger than that of any state for D with exactly one B-split.

6. Consider the tangles T_1 (left) and T_2 (right) shown:



(a) Calculate $\langle T_1 \rangle$.

(b) Calculate $\langle T_2 \rangle$. (I suggest breaking this into 2 tangles first, noting that you calculated the bracket of the rational tangles -1 and -2 along the way in your answer for part (a), and then reassembling the pieces.)

(c) Combining parts (a) and (b), compute the bracket polynomial of the following knot: