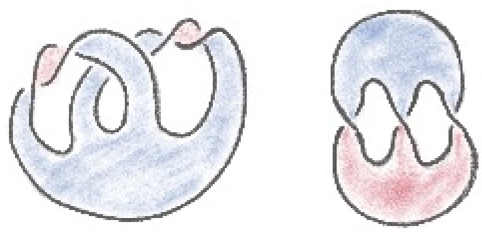
Math 309: Introduction to knot theory

Assignment 6, due Monday November 23 by 11:59 pm.

Exercises.

1. Show that the Alexander polynomial of a splittable link is always zero.

2. Using a series of pictures, show that the following two surfaces are equivalent:



(Hint: Try sliding one band over the other.)

3. There are 3 non-alternating 8-crossing knots in the table. For each of these, use Seifert's algorithm to find a Seifert surface and compute the genus of this surface. Then, in each case, check if the genus you calculated is equal to the Seifert genus of the knot in question. (You should make use of The Knot Atlas or similar online resources for this second part of the exercise.)

4. Prove that the following knot has trivial Alexander polynomial:

Problems

5. Two infinite families of Alexander polynomials.

(a) Let $T_{n,2}$, $n \in \mathbb{Z}$, be a 2-stranded torus knot (this family includes 3_1, 5_1...). Give a formula for the Alexander polynomial $\Delta_{T_{n,2}}$ for all n.

(b) Let K_n denote the n^{th} twist knot (this family includes 4_1, 5_2...) Give a formula for the Alexander polynomial Δ_{K_n} for any n.

6. Compute the linking form for the knot 6_3 (using the diagram given in the table). Using this, calculate the Alexander polynomial.