Math 309: Introduction to knot theory

Assignment 7, due Monday December 7 by 11:59 pm.

Exercises.

1. Consider the braid $\beta = \sigma_2^3\sigma_1\sigma_2^{-1}\sigma_1$. Identify the knot in the table obtained by taking the braid closure $\bar{\beta}$.

2. Show that the braids σ_1^3 and $(\sigma_1\sigma_2)^2$ are Markov equivalent by giving an explicit list of Markov moves and braid moves.

3. Pick a non-alternating 8 crossing knot and prove that its braid index is at most 3.

4. The braid groups are not abelian in general: find braids β_1 and β_2 for which $\beta_1\beta_2$ and $\beta_2\beta_1$ are not equivalent. (To prove inequivalence, try considering the associated permutations.)

Problems

5. A group is a set equipped with an associative binary operation (often called multiplication) and a special element called the unit. A simple example is given by $(\mathbb{Z}, +)$ where $\mathbb{Z}$ denotes the integers and $+$ is the usual addition.

(a) Give the definition of a group homomorphism from a group G_1 to a group G_2.

(b) If B_2 is the two-strand braid group, show that there is a homomorphism from B_2 to $\mathbb{Z}$. Similarly, show that there is a homomorphism from $\mathbb{Z}$ to B_2.

(c) Give the definition of a group isomorphism between groups G_1 and G_2. Are B_2 and $\mathbb{Z}$ isomorphic groups? Why or why not?

6. Using Artin combing, show that each of the following braids is non-trivial in the 3-strand braid group.

(At some stage, in your argument, I anticipate an appeal to Problem 5!)