LECTURE 1: PROJECTIONS.

CONSIDER POINTS $(x,y,z) \in \mathbb{R}^3$ SUBJECT TO:

$$[\bigstar] \qquad x^2+y^2=1 \quad \text{AND} \quad z=0$$

THESE ARE POINTS ON A CIRCLE IN THE xy-PLANE; THEY FORM A CLOSED LOOP IN $\mathbb{R}^3$.

__DEF__ A "KNOT" IS A CLOSED CURVE IN $\mathbb{R}^3$ THAT DOES NOT INTERSECT ITSELF.

NOTE PERIODICITY IN THIS DEFINITION.
THIS IS EASIER TO SEE, IN OUR EXAMPLE, IF WE PARAMETRIZE THE CIRCLE:

$$t \longmapsto (\cos t, \sin t, 0)$$

$x(t) = \cos t$ $\quad$ $y(t) = \sin t$ $\quad$ $z(t) = 0$

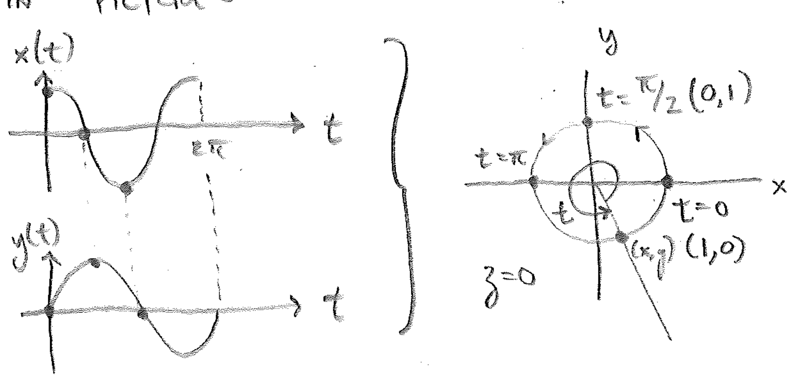
AND

$$x^2+y^2 = \cos^2 t + \sin^2 t = 1$$

TRIG!

SO THAT $[\bigstar]$ IS SATISFIED.

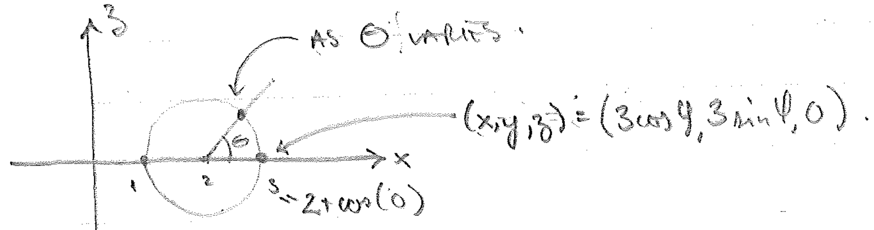
IN PICTURES



NOW CONSIDER THE SURFACE

$$x = (2+\cos\theta)\cos\varphi$$
$$y = (2+\cos\theta)\sin\varphi$$
$$z = \sin\theta \qquad \text{WHERE } 0 \le \theta, \varphi \le 2\pi$$

YOU'VE ENCOUNTERED THIS AS A "SURFACE OF REVOLUTION"

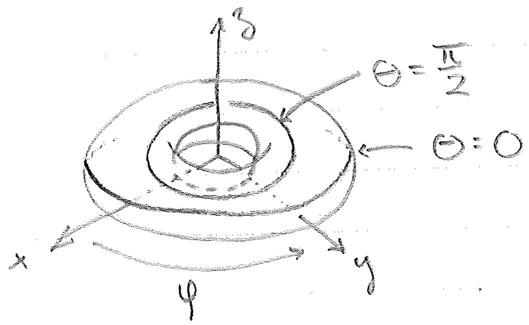


(z IS THE "AXIS OF REVOLUTION")

(PICTURE: FOR $\theta = 0$, $z = \sin(0) = 0$).

THIS SURFACE IS A "TORUS"; CALL IT T

NOTE THAT $T \subset \mathbb{R}^3$ (A SUBSET).



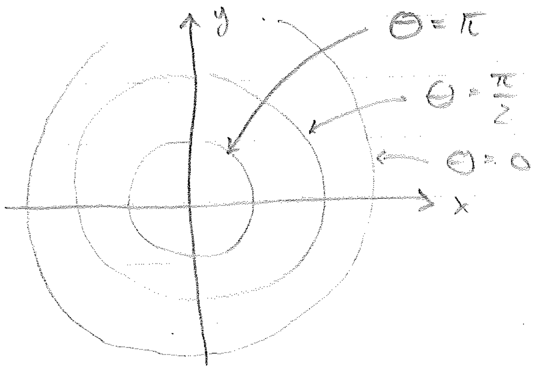
NOTE: FOR $\theta = \pi$, $\varphi = t$ WE GET:

$$(x,y,z) = (\cos t, \sin t, 0)$$

SO OUR KNOT IS A SUBSET OF T.
(IN RED)

OTHER SUBSETS: $\theta = \frac{\pi}{2}, \varphi = t$
(IN BLUE). $\theta = 0, \varphi = t$

NOW LET'S PROJECT TO THE xy-PLANE
(VIEW FROM TOP; SET $z=0$).

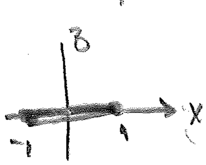


WE'D LIKE TO VIEW THESE AS 3
DIFFERENT PROJECTIONS OF THE SAME KNOT

__DEF__ TWO KNOTS ARE EQUIVALENT IF ONE
MAY BE CONTINUOUSLY DEFORMED
TO THE OTHER, IN PARTICULAR, AT
ANY STAGE OF THIS DEFORMATION
YOU SHOULD STILL HAVE A KNOT.

SOME PROJECTIONS ARE __BAD__:

__EXP__ PROJECTING $(\cos t, \sin t, 0)$ TO xz-PLANE



.. LOSE TOO MUCH INFO

__EXP__ FOR OUR EXAMPLE, ONLY PROJECTION
PLANES CONTAINING z-AXIS CAUSE
THIS ISSUE ... SO, MOST PROJ'S OK!

NOW CONSIDER A LARGER CLASS OF KNOTS:

$$\theta \longmapsto pt\,, \quad \varphi \longmapsto qt \qquad p,q \in \mathbb{Z}$$

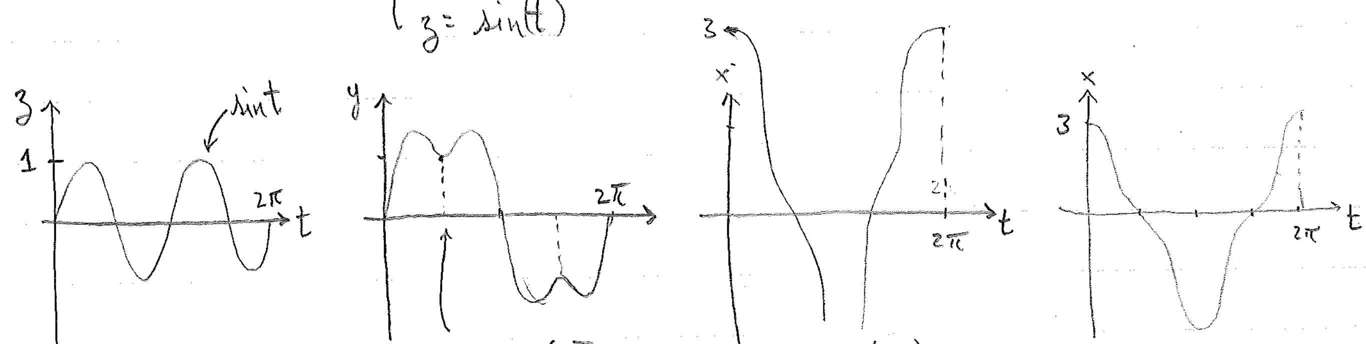
EXC CHECK THAT WE'VE BEEN LOOKING AT $p=0, q=1$.

$$\Big(\underbrace{(2+\cos(pt))\cos(qt)}_{x}\,,\ \underbrace{(2+\cos(pt))\sin(qt)}_{y}\,,\ \underbrace{\sin(pt)}_{z}\Big)$$

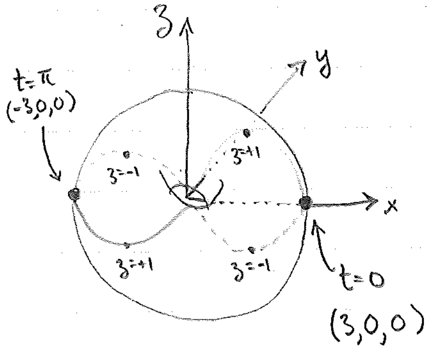
FOR EACH p,q RELATIVELY PRIME, THIS IS A KNOT.

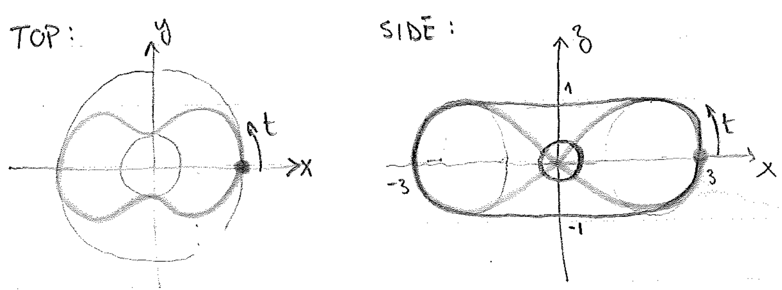
EXP $p=2,\ q=1$.

$$\begin{cases} x = (2+\cos(2t))\cos(t) \\ y = (2+\cos(2t))\sin(t) \\ z = \sin(t) \end{cases}$$



$2+\underbrace{\cos(2\tfrac{\pi}{2})}_{-1}=1$ $\qquad$ $2+\underbrace{\cos(2\cdot 0)}_{1}=3$





STILL THE SAME KNOT... ... BUT NEW ISSUE IN THIS PROJECTION.

BUT UNLIKE THE $(\cos t, \sin t, 0)$ EXAMPLE, WHICH HAD A *BAD* PROJECTION, THIS ONE CAN BE FIXED IF WE KEEP TRACK...