Math 308: Introduction to tilings

Assignment 1, due September 24 by 11:59 pm.

Exercises.

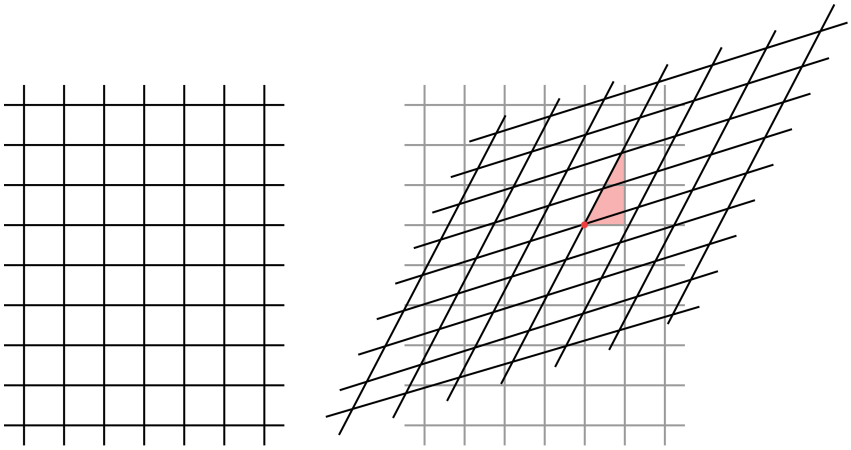
1. Consider the "honeycomb tiling" of the plane by regular hexagons, seen in Lecture 2. By verifying that the definition holds, check that this is indeed a tiling. **Hint:** Start by giving a definition of *countability*.

2. Show that any triangle may be used as a prototile for an edge-to-edge monohedral tiling of the plane.

3. Find a tiling of the plane by congruent (but not regular) octagonal tiles such that no tile vertex has angle π (that is, you are not allowed to use a square with subdivided edges!). Is it possible to choose your tile so that it is convex? Why or why not?

Problems

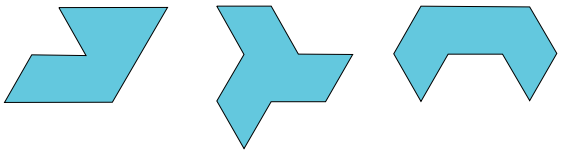
5. In each of the following tilings of the plane, assume that the tiles are identical and each have unit area:



To move from one tiling to another, a composition of a shear and a rotation is required (the centre of rotation is indicated by the red dot). Find each these, and give an explicit transformation of the plane that takes one tiling to the other. Are the two tilings equal in the sense defined in class? Why or why not?

6. Heptiamonds are discussed on Page 21 of Grünbaum and Shephard.

(a) For each of the following three heptiamonds, find a monohedral tiling with the given tile as prototile.



Hint: Think about patches.

(b) Prove that there is no monohedral tiling with prototile