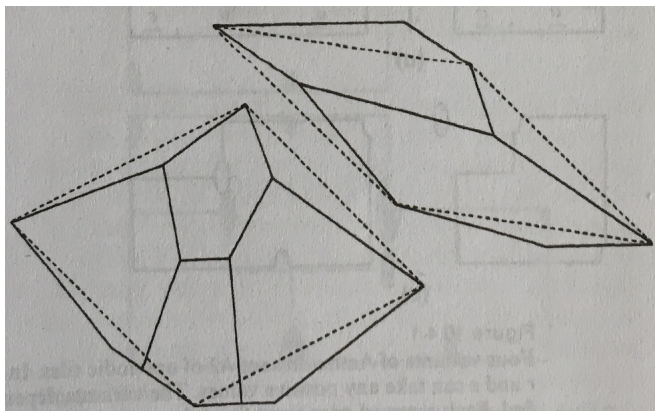


**Math 308: Introduction to tilings**  
**Assignment 6, due December 7 by 11:59 pm.**

**Note:** while the official deadline is December 7, in order to facilitate variation in exam schedules, this assignment may be submitted up to 7 days after the deadline (until December 14 at 11:59 pm).

**Exercises.**

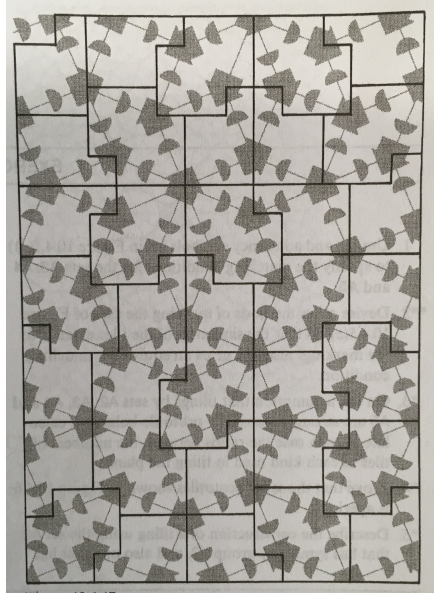
1. Let  $\{T_1, T_2\}$  be a tile set admitting a periodic dihedral tiling  $\mathcal{T}$ . Prove that the relative density of  $T_1$  tiles to  $T_2$  tiles occurring in  $\mathcal{T}$  must be a rational number.
2. In class I gave an example of a kite tile (contributing to the count  $k_3$ ) with the decomposition process applied twice (to produce a patch consisting of tiles contributing to the counts  $k_1$  and  $d_1$ ). For this problem, showing your steps, you should repeat this process, this time applying the decomposition process three times (that is, you should start with a kite contributing to the count  $k_4$ ). Repeat this process a second time, this time starting from a dart (contributing to the count  $d_4$ ) and working backwards.
3. Consider the following recomposition of Penrose rhombs:



Show that, for appropriately chosen parameters, this recomposition gives rise to an aperiodic set of 6 convex (non-regular) pentagons.

## Problems

4. This problem pertains to material on pages 556–557 of Grünbaum and Shephard. Consider the following recomposition:



Assuming that the set  $A_4$  is aperiodic, use this recomposition sequence to prove that the set  $A_5$  is aperiodic.

5. This final problem of the course requires a **short essay answer**: Starting from the tiling of your choice, carefully single out and describe/establish/prove the existence of a property of this tiling that you learned about over the course of this term (and, hopefully, find interesting). **Guidelines:** Because of the differences in how you are all submitting your coursework, I do not want to set explicit page limits for this question. However, if you submit only a single page you should expect a low mark; similarly, it is unreasonable for me to ask you to submit 10+ pages of work towards answering this question. I think that a reasonable range to aim for is on the order of 4 to 6 pages, again, depending heavily on the technology you have been using to prepare your work. **If you are unsure, you should feel free to discuss the tiling/property that you plan to prepare as a solution to this problem with me—well ahead of the submission deadline!**