WHY IS \((a+b)^2 \neq a^2 + b^2\)?

(HERE \(a, b\) ARE NON-NEGATIVE INTEGERS)

REASON: BEDMAS

\[
\begin{aligned}
\text{BRACKETS} & \quad \text{EXPONENTS} \\
\text{DIV/MULT} & \quad \text{ADD/SUB}
\end{aligned}
\]

THAT IS \((a+b)^2 = a^2 + 2ab + b^2\)

\[
\Rightarrow \quad [(a+b)^2] - [a^2 + b^2] = 2ab
\]

ЭТО "CROSS TERM"

SO: AS LONG AS NEITHER OF \(a\) OR \(b\) IS ZERO THEN \(2ab \neq 0\) HENCE THE QUANTITIES DIFFER.

THIS IS "CORRECT" BUT TO ME IT FEELS LIKE SAYING "BECAUSE IT JUST IS".

GOAL: TRY TO FIND GEOMETRIC INTERPRETATION.

\[
\begin{aligned}
A &= \text{AREA OF A SQUARE SIDE LENGTH } a+b \\
A' &= \text{AREA OF A SQUARE SIDE LENGTH } a
\end{aligned}
\]

CONSTRUCTION: \(A' < A\)
WE KNOW THAT \( A = (a+b)^2 \) TO GET \( A' \), WHAT DO WE NEED TO REMOVE?

4 TRIANGLES OF AREA \( \frac{1}{2}ab \)

FIRST DECOMP. \( \Rightarrow \)
\[
A' = A - 4 \left( \frac{1}{2}ab \right)
\]
\[
r^2 = (a+b)^2 - 2ab
\]
OUR CROSS TERM

NOW WHAT IF WE SLICE UP THE ORIGINAL SQUARE DIFFERENTLY?

\[
B = b^2 \text{ (AREA OF A SQUARE)}
\]
\[
C = a^2 \text{ (AREA OF A SQUARE)}
\]

SECOND DECOMP. \( \Rightarrow \)
\[
A = B + C + 4 \left( \frac{1}{2}ab \right)
\]
\[
A = b^2 + a^2 + 2ab
\]

\[
0^2 \quad b^2 + a^2 = A - 2ab
\]
\[
r^2 = (a+b)^2 - 2ab
\]
SAME QUANTITY

\[
\text{SO} \quad r^2 = b^2 + a^2.
\]

WE HAVE LEARNED:

1. WE GAVE A PROOF THAT THAT IS, THE PYTHAGOREAN THEOREM.
   (SHOWED \( B + C = A' \iff b^2 + a^2 = r^2 \))

2. FOUND THAT THE CROSS TERM MAY BE VIEWED AS AN AREA, AND THAT AREA IS THE DIFFERENCE BETWEEN \( A \) AND \( A' \),
   \( A - A' = 2ab \iff A - (B + C) = 2ab \iff \left[ (a+b)^2 \right] - \left[ b^2 + a^2 \right] = 2ab \).
WHAT IF WE TILED THE PLANE WITH EACH OF THESE PATTERNS?

QUESTION: ARE THESE DIFFERENT?
WHAT DOES DIFFERENT EVEN MEAN IN THIS CONTEXT?