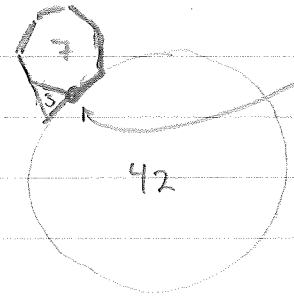
LECTURE # 15

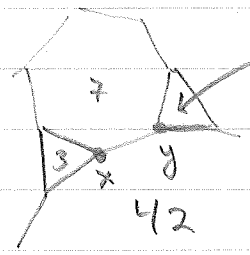
WE SAW THAT $\frac{1}{3}+\frac{1}{7}+\frac{1}{42}=\frac{1}{2}$ CORRESPONDED TO THE "VERTEX SPECIES"



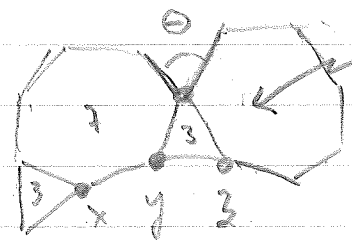

$$\underbrace{\frac{1}{3}\pi+\frac{5}{7}\pi+\frac{40}{42}\pi}=2\pi$$

$$\frac{14+30+40}{42}\pi \qquad 7\cdot 3\cdot 2$$

TRY TO EXTEND THIS TO A "UNIFORM" TILING, WHERE EVERY VERTEX IS OF THIS TYPE (AND ALL EDGES HAVE SAME LENGTH)...




FORCED; ANGLE SUM WORKS AT VERTEX $y$




FORCED; ANGLE SUM WORKS AT VERTEX 7.

WHAT IS $\Theta$? IN ORDER TO GET A $\{3,7,42\}$ VERTEX IT HAS TO BE $\frac{40}{42}\pi=\frac{20}{21}\pi$

THAT IS, WE REQUIRE A $\{3,7,7,42\}$ VERTEX.
IS THIS IMPOSSIBLE (SEE HOMEWORK), BUT ALSO

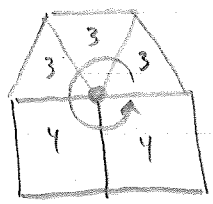
$$\frac{1}{3}\pi+\frac{5}{7}\pi+\frac{40}{42}\pi=2\pi \iff \frac{1}{3}\pi+\frac{5}{7}\pi+\frac{5}{7}\pi+\frac{40}{42}\pi>2\pi$$

$$\Theta=2\pi-\frac{1}{3}\pi-\frac{10}{7}\pi=\frac{42-7-30}{21}\pi=\frac{5}{21}\pi\neq\frac{1}{3}\pi,\ \frac{5}{7}\pi,\ \frac{20}{21}\pi$$

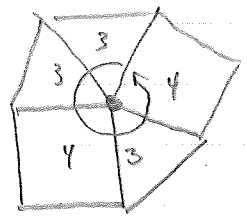
SO WHILE THERE ARE 17 VERTEX SPECIES, AND 21 VERTEX TYPES, THERE ARE ONLY 11 UNIFORM TILINGS.

<u>EXP</u> $\{3,3,3,4,4\}$ VERTEX SPECIES #3 FROM G&S.

TWO VERTEX TYPES:

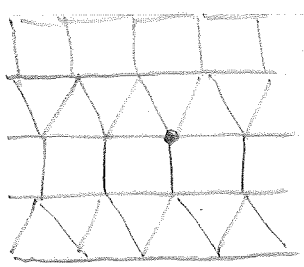


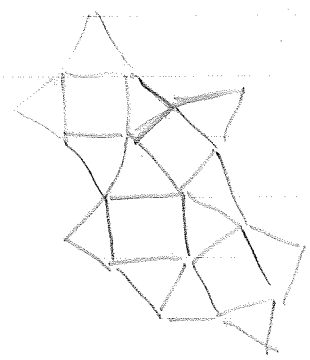

3.3.3.4.4




3.3.4.3.4

TILINGS EXIST IN BOTH CASES (THIS PART IS CONSTRUCTIVE):





G&S NOTATION $(3^3.4^2)$ $(3^2.4.3.4)$

THESE 11 TILINGS — SEE G&S 2.1.3 ON PAGE 59 — ARE "UNIFORM" IN THE FOLLOWING SENSE:

FOR ANY TWO VERTICIES $x, y \in \mathcal{T}$ THERE IS A SYMMETRY $g \in S(\mathcal{T})$ SO THAT $g(x) = y$.

TERMINOLOGY: EVERY VERTEX IS IN THE SAME "TRANSITIVITY CLASS."

RECALL THAT WE SAW A MONOHEDRAL TILING $\mathcal{T}$ IS ISOHEDRAL IF $T_1, T_2 \in \mathcal{T}$ CAN BE RELATED BY $gT_1 = T_2$ FOR $g \in S(\mathcal{T})$.

NOTE THAT SUB-OBJECTS IN A GIVEN $\mathcal{T}$ MAY OR MAY NOT BE RELATED BY A SYMMETRY. THIS CAN BE PHRASED AS AN EQUIVALENCE RELATION:

GIVEN $T_1, T_2 \in \mathcal{T}$ DEFINE $T_1 \sim T_2$ WHEN THERE IS A $g \in S(\mathcal{T})$ SO THAT $g(T_1) = T_2$.

REFLEXIVITY: $T \sim T$ FOR ALL $T \in \mathcal{T}$
SYMMETRY: $T_1 \sim T_2 \iff T_2 \sim T_1$
TRANSITIVITY: $T_1 \sim T_2$ AND $T_2 \sim T_3$ IMPLIES $T_1 \sim T_3$.

AN "EQUIVALENCE CLASS" IS THE SET OF ALL SUB-OBJECTS (TILES, IN THIS CASE) RELATED BY THIS EQUIVALENCE. SO

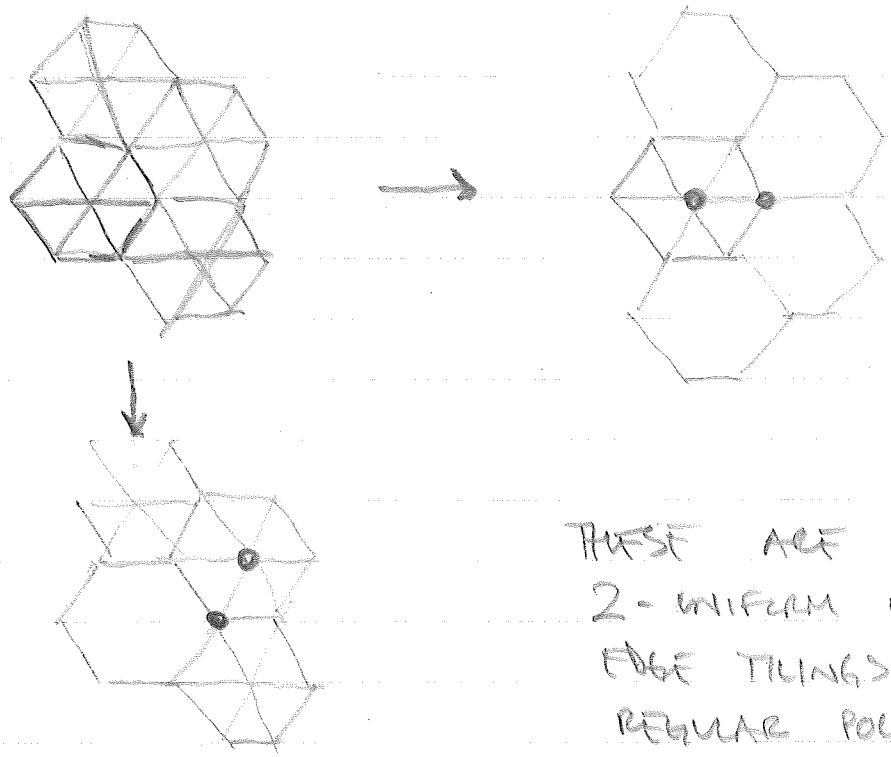
★ A MONOHEDRAL $k$-ISOHEDRAL TILING HAS <u>EXACTLY</u> $k$ EQUIVALENCE CLASSES OF TILES ★

A "UNIFORM TILING" (EDGE TO EDGE AND BY REGULAR POLYGONS) HAS ONE EQUIVALENCE OF <u>VERTEX</u> (i.e THE VERTEX TYPE).

INTERESTING GENERALIZATION: $k$-UNIFORM SO THAT THERE ARE $k$-EQUIVALENCE CLASSES OF VERTICES.

THIS ALLOWS US TO PICK $k$ VERTEX TYPES FROM OUR POSSIBLE 21, AND TRY TO PIECE THEM INTO A TILING.

EXP



WHY?

THESE ARE DISTINCT 2-UNIFORM EDGE-TO-EDGE TILINGS BY REGULAR POLYGONS.