WHAT IS A TILING?

LET Ψ BE THE (EUCLIDEAN) PLANE. THIS CAN BE IDENTIFIED WITH $\mathbb{R}^2$ (THE REAL SPACE)

A "TILING" $\mathcal{T}$ OF THE PLANE IS A COUNTABLE FAMILY OF CLOSED SETS

$$\mathcal{T} = \{T_1, T_2, T_3 \ldots\}$$

THAT COVER Ψ WITHOUT OVERLAPS OR GAPS IN SYMBOLS:

$$\Psi = \bigcup_{i=1}^{\infty} T_i \quad \text{AND} \quad T_i \cap T_j = \begin{cases} \emptyset \text{ OR} \\ \text{PART OF THE BOUNDARY OF BOTH } T_i \text{ AND } T_j \end{cases}$$

BORING EXAMPLE

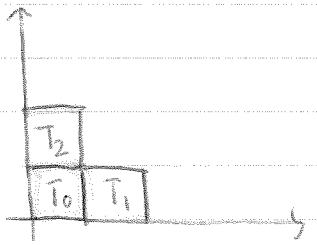
$T_1 = \Psi$. Ψ IS TILED BY JUST ONE TILE - THE ENTIRE PLANE.

↑ SATISFIES ALL BUT THE CLOSED CONDITION

LESS BORING

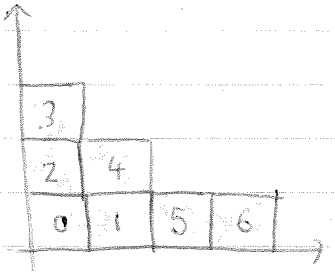
T_0 = UNIT SQUARE

USE THIS TO BUILD THE (FAMILIAR) CARTESIAN PLANE.



T_1 is T_0 TRANS TO RIGHT 1 UNIT

T_2 is T_1 TRANS ~~TO THE LEFT~~ UP 1 UNIT



(i) EACH $T_i = \square$; $\bigcup_{i=0}^{\infty} T_i$ = 1ST QUADRANT

(ii) THIS ENUMERATION TAKES POSITIVE INTEGERS n TO TILES

$$n \longmapsto T_n$$

(iii) BY ROTATING, WE CAN USE THIS TO TILE ALL OF $\mathbb{P}$; A UNION OF COUNTABLE SETS IS COUNTABLE

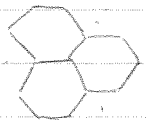
$$T_1 \cap T_{90} = \emptyset$$

$$T_0 \cap T_1 = \boxed{T_0 \; T_1}$$

RMK TILES ~~EATHER~~ EITHER DO NOT MEET OR THEY MEET ON BOUNDARIES, LETS WRITE ∂T FOR "BOUNDARY OF T".

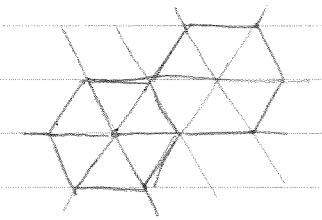
THE T_i ARE THE "TILES" OF A GIVEN TILING $\mathcal{T}$.

EXP (HONEYCOMB TILING)



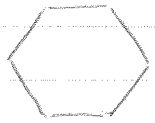
TILING BY T_i = REGULAR HEXAGONS

(HEXAGONAL LATTICE)



TILING BY EQUILATERAL TRIANGES. $T_i = \triangle$

RMK THE T_i IN A GIVEN $\mathcal{T}$ NEED NOT BE ALL THE SAME! TRY MIXING THE LAST TWO EXAMPLES AND USE

$T_i =$ OR 

SEE ALSO G&S OR EXAMPLE FROM END OF LAST DAY.

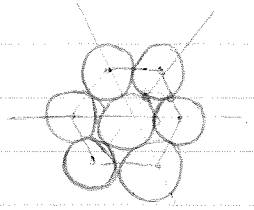
USEFUL GADGET:
A PATCH IS A SUBSET OF TILES IN A GIVEN TILING
USUALLY WE'LL ASSUME A PATCH IS CONNECTED.

LAST TIME WE SAW TWO PATCHES FOR TILING $\mathcal{P}$:

- TRIANGLE TILES ARE ALL THE SAME
- THE FIRST HAS 1 SQUARE OF SIDE $\sqrt{a^2+b^2} = r$
- THE SECOND HAS 2; ONE OF SIDE a, ONE OF SIDE b

RMK G&S OBSERVE THAT A TILING IS AN OBJECT THAT IS BOTH A "PACKING" AND A "COVERING"
↑ NO OVERLAP CONDITION ↑ NO GAPS CONDITION.

EXP LET'S START WITH A PACKING OF $\mathcal{P}$ BY DISKS:



NO OVERLAPS, BUT THERE ARE GAPS.

TO TURN THIS INTO A TILING WE NEED TO ADD TILES TO FILL THESE GAPS

SO IN THIS TILING $T_i =$ OR