LECTURE 32

LAST TIME

\[ \Theta = \frac{\pi}{3} (36^\circ) \]

This is related to Houdini's construction by decorations:

So that:

- Star
- Then pentagons
- Then darts
- Then kites

\[ nK + mD = A\text{ PATCH} \]

Consisting of n kites and m darts

We saw that 2K + D gives a new (larger) kite:

\[ T = \frac{1 + \sqrt{5}}{2} \]

"Golden Nuage"
This gives rise to a new composition sequence giving a similarity of a kite-dart tiling (assuming we've constructed one).

Let \( k_n \) be the number of kites after \( n \) compositions.

Let \( d_n \) be the number of darts after \( n \) compositions.

Note that we compose 2 kites + \( \left( \frac{1}{2} + \frac{1}{2} \right) \) darts to get a kite.

\[ k_{n+1} = 2k_n + d_n \]

And we compose 1 kite and \( \left( \frac{1}{2} + \frac{1}{2} \right) \) darts to get a new dart.

\[ d_{n+1} = k_n + d_n \]

Consider the ratio:

\[ \frac{k_{n+1}}{d_{n+1}} = \frac{2k_n + d_n}{k_n + d_n} = \frac{1 + \frac{k_n}{d_n}}{1 + \frac{k_n}{d_n}} \]

If this ratio is \( x \), notice that when \( n \to \infty \)

\[ x = \frac{1 + 2x}{1 + x} \]

\[ \frac{1 + x}{1 + x} x = 1 + 2x \]

\[ x^2 - x - 1 = 0 \]

The positive root of \( x^2 - x - 1 = 0 \) is our kite-dart ratio (in the limit):

\[ x = \frac{1 + \sqrt{5}}{2} \]
In your homework, you will see that a periodic difference thing must have rational ratio.

So the fact that $x = \frac{1 + \sqrt{3}}{2}$, which is irrational, shows up guarantees non-periodicity.

Here's an example: