Math 427/527: algebraic topology

Homework 2, due Wednesday March 2 by 5:00 pm.

1. Let $f\colon S^n \to S^n$ be a map where $n > 0$ and for which $f^{-1}(y) = \{x_1, \dots, x_m\}$ (compare Lecture 10).

(i) By carefully choosing an appropriate CW structure on S^n, calculate $H_n(S^n, S^n \smallsetminus f^{-1}(y))$.

(ii) Let k_i be the map induced by inclusion

$$(U_i, U_i \smallsetminus x_i) \to (S^n, S^n \smallsetminus f^{-1}(y))$$

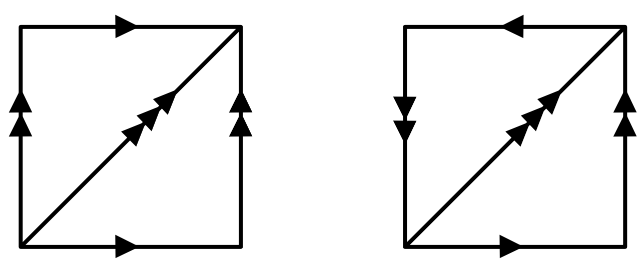
(where U_i is a neighbourhood of x_i) and let p_j be the map induced by projection

$$(S^n, S^n \smallsetminus f^{-1}(y)) \to (S^n, S^n \smallsetminus x_j)$$

Show that $p_j \circ k_i$ vanishes when $i \neq j$.

2. Let X be a CW complex. Using the mapping telescope construction (Hatcher, page 138) complete the last step of the proof given in class during Lecture 11: show that $\widetilde{H}_n(X^{(n+1)}) \cong \widetilde{H}_n(X)$.

3. Consider the following CW complexes:



Calculate the homology groups for each of the above using the cellular chain complex, being sure to give detailed calculations of the degrees of the maps required and the corresponding coefficients in the differentials (compare the example at the end of Lecture 8).

4. Let $\mathbb{R}\mathrm{P}^n$ be the space of lines through the origin in $\mathbb{R}^{n+1}$.

(i) Using the fact that $* \subset \mathbb{R} \subset \mathbb{R}^2 \subset \cdots \subset \mathbb{R}^k \subset \cdots \subset \mathbb{R}^{n+1}$ by restriction to the first $k \geq 0$ coordinates, say, find an explicit CW structure on $\mathbb{R}\mathrm{P}^n$.

(ii) Extract the gluing maps from your work in (i), and use this to give a detailed calculation of the groups $H_*(\mathbb{R}\mathrm{P}^n; \mathbb{Z})$ for any $n \geq 0$ using the cellular chain complex.