Math 427/527: algebraic topology
Homework 3, due Wednesday March 30 by 5:00 pm.

Read Hatcher's Section 3.2.

1. Let α and β be an appropriate choice of dual basis for $H^1(S^1 \times S^1) \cong \mathrm{Hom}(H_1(S^1 \times S^1), \mathbb{Z})$. Assume coefficients in $\mathbb{Z}$ throughout.

(i) If $\alpha \cup \beta = 1$ in $H^2(S^1 \times S^1)$, check that $\beta \cup \alpha = -1$ by carefully chasing through the definition given in class.

(ii) Part (ii) is the essential check that $H^*(S^1 \times S^1)$ agrees with the alternating algebra on α and β as a ring. Assuming this ring isomorphism, calculate the ring $H^*(\Sigma_g)$ where Σ_g is a closed, orientable surface of genus $g > 1$ (this is Exercise 1 in Hatcher's Section 2.3).

2. Prove that $S^5 \times S^2$ and $S^7 \vee S^5 \vee S^2$ are not homotopy equivalent.