Math 309: Introduction to knot theory Assignment 7, due Thursday April 13 by 11:59 pm.

1. Show that the braids σ_1^3 and $(\sigma_1 \sigma_2)^2$ are Markov equivalent by giving an explicit list of Markov moves and braid moves.

2. The braid groups are not abelian in general: find braids β_1 and β_2 for which $\beta_1\beta_2$ and $\beta_2\beta_1$ are not equivalent. (To prove inequivalence, try considering the associated permutations.)

3. A group is a set equipped with an associative binary operation (often called multiplication) and a special element called the identity (or neutral element). A simple example is given by $(\mathbb{Z}, +)$ where \mathbb{Z} denotes the integers and + is the usual addition.

(a) Give the definition of a group homomorphism from a group G_1 to a group G_2 .

(b) If B_2 is the two-strand braid group, show that there is a homomorphism from B_2 to \mathbb{Z} . Similarly, show that there is a homomorphism from \mathbb{Z} to B_2 .

(c) Give the definition of a group isomorphism between groups G_1 and G_2 . Are B_2 and \mathbb{Z} isomorphic groups? Why or why not?

4. Using Artin combing, show that each of the following braids is non-trivial in the 3-strand braid group.



(At some stage, in your argument, I anticipate an appeal to Problem 3!)