Math 427/527: algebraic topology
Homework problem for Lecture 10.

Let $\mathbb{R}\mathrm{P}^n$ be the space of lines through the origin in $\mathbb{R}^{n+1}$.

(i) Using the fact that $* \subset \mathbb{R} \subset \mathbb{R}^2 \subset \cdots \subset \mathbb{R}^k \subset \cdots \subset \mathbb{R}^{n+1}$ by restriction to the first $k \geq 0$ coordinates, say, find an explicit CW structure on $\mathbb{R}\mathrm{P}^n$.

(ii) Extract the gluing maps from your work in (i), and use this to give a detailed calculation of the groups $H_*(\mathbb{R}\mathrm{P}^n; \mathbb{Z})$ for any $n \geq 0$ using the cellular chain complex.