Math 427/527: algebraic topology

Homework problem for Lecture 2.

Poincaré was first to realize that homology was weaker than homotopy, and he revised his now-famous conjecture accordingly.

(a) Show that the universal cover of the special orthogonal group $SO(3)$ may be identified with the unit quaternions, and is therefore homeomorphic to the three-sphere S^3.

(b) Let I be the icosahedral group. Define an action of I on $SO(3)$, and determine the group $\tilde{I}$ acting on S^3, associated with the cover in part (i).

(c) Show that $\tilde{I}$ is a perfect group, and conclude that there exists a topological space, locally homeomorphic to $\mathbb{R}^3$, that has non-trivial fundamental group but trivial first homology (assuming that this latter group is the abelianization of the former).