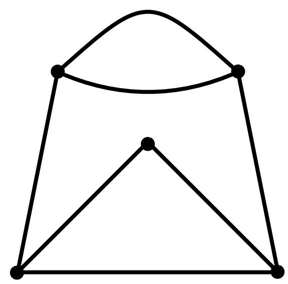
Math 427/527: algebraic topology

Homework problem for Lecture 4.

In Lecture 4 (January 20) we developed a scheme, as a means of providing motivation and generating intuition, for calculating the first homology group of a graph. For this problem, consider the following graph Γ:



(a) Calculate the homology $H_1(\Gamma; \mathbf{Z})$ according to our scheme from Lecture 4, in particular, you should give a basis in terms of cycles. Show all of your work.

(b) In part (a), you needed to choose orientations on each edge. Prove that your answer does not depend on this choice.

(c) By finding an appropriate presentation for $\pi_1(\Gamma)$, prove that

$$H_1(\Gamma; \mathbf{Z}) \cong \frac{\pi_1(\Gamma)}{\big[\pi_1(\Gamma), \pi_1(\Gamma)\big]}$$

(There are many ways to go about this, but here is what I am asking so that this ties to the preceding work: You should pick a basis for $\pi_1(\Gamma)$ using, by abuse, the same conventions that you used to solve (a), so that when you quotient by the commutator subgroup you land on precisely the presentation you gave above.)