Math 427/527: algebraic topology

Homework problem for Lecture 7.

Let Γ be a cyclic group of order 2 and consider the group ring $\mathbb{Z}\Gamma$. Consider the chain complex given by $C_i = \mathbb{Z}\Gamma$, for $i \geq 0$, and

$$d_i(\gamma) = \begin{cases} (1+x)\cdot\gamma & \text{for } i \text{ even} \\ (1-x)\cdot\gamma & \text{for } i \text{ odd} \end{cases}$$

where x is a generator for Γ.

(a) A chain complex is called *acyclic* if the homology vanishes in all degrees above 0. Prove that C_* is acyclic.

(b) Find an augmentation for C_* so that $H_0(C_*)$ vanishes.