Math 427/527: algebraic topology
Homework problem for Lecture 9.

Let $f\colon S^n \to S^n$ be a map where $n > 0$ and for which $f^{-1}(y) = \{x_1, \ldots, x_m\}$.

(i) By carefully choosing an appropriate CW structure on S^n, calculate $H_n(S^n, S^n \smallsetminus f^{-1}(y))$.

(ii) Let k_i be the map induced by inclusion

$$(U_i, U_i \smallsetminus x_i) \to (S^n, S^n \smallsetminus f^{-1}(y))$$

(where U_i is a neighbourhood of x_i) and let p_j be the map induced by projection

$$(S^n, S^n \smallsetminus f^{-1}(y)) \to (S^n, S^n \smallsetminus x_j)$$

Show that $p_j \circ k_i$ vanishes when $i \neq j$.