Math 308: Introduction to tilings

Assignment 1, due Friday September 29 by 11:59 pm.

1. Show that any triangle may be used as a prototile for an edge-to-edge monohedral tiling of the plane.

2. Find a tiling of the plane by congruent (but not regular) octagonal tiles such that no tile corner has angle π (that is, you are not allowed to use a square with subdivided edges!). Is it possible to choose your tile so that it is convex? Why or why not?

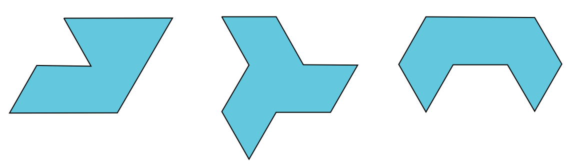
3. Consider the "honeycomb tiling" of the plane by regular hexagons, seen in lectures.

(a) By verifying that the definition holds, check that this is indeed a tiling. **Hint:** Start by giving a definition of *countability*.

(b) In Lecture 3, I stated the non-obvious fact that non-negative integer solutions (x, y) to the equation $x^2 + xy + y^2 = n$ (where n is a natural number) gave rise to similarities of the honeycomb tiling. Taking this fact for granted, and assuming that the pair $(1, 0)$ corresponds to a tiling by regular hexagons that can be inscribed in a unit circle, describe the tilings corresponding to $(1, 1)$ and to $(2, 1)$. Be sure to explain your answer. **Hint:** Everything should be describable within a circle of radius 3 centred at the origin.

4. Heptiamonds are discussed on Page 21 of Grünbaum and Shephard.

(a) For each of the following three heptiamonds, find a monohedral tiling with the given tile as prototile.



Hint: Think about patches.

(b) Prove that there is no monohedral tiling with prototile