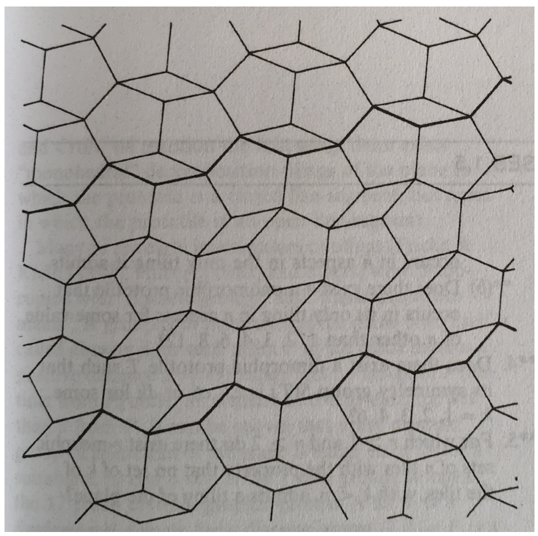
Math 308: Introduction to tilings

Assignment 2, due October 14 by 11:59 pm.

1. Suppose that a tiling $\mathcal{T}$ admits a rotational symmetry that is unique, in the sense that the centre of this rotation is the only point in the plane about which a rotation gives rise to a symmetry of the tiling. Prove that $S(\mathcal{T})$ does not contain a translation. (Note that this problem is related to our discussion of symmetries from Lecture 6.)

2. Consider the following periodic tiling from Grünbaum & Shephard:



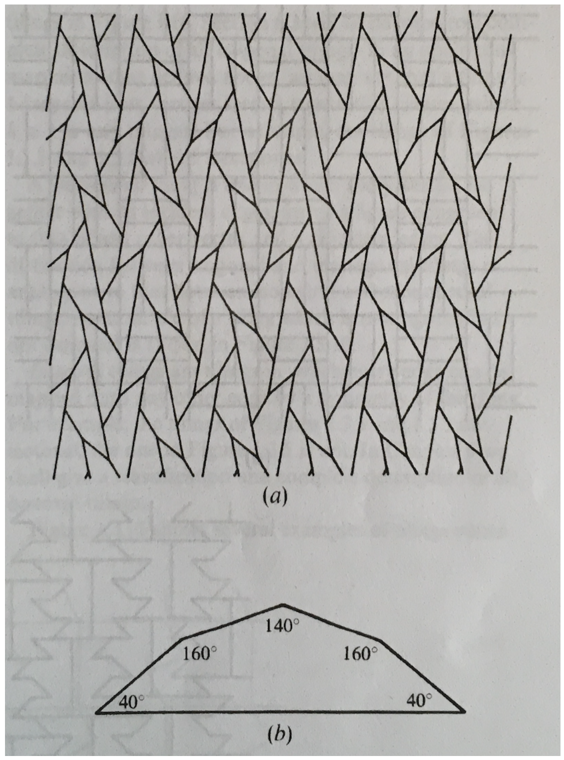
Prove that this tiling is periodic, making your choice of vectors $\vec{a}$ and $\vec{b}$ explicit. Next, find the $P(\vec{a}, \vec{b})$ associated with the $\vec{a}$ and $\vec{b}$ found, and give a $\widehat{P}(\vec{a}, \vec{b})$ determined by your choices. Show all your work.

3. Let T be a unit square admitting the standard square tiling of the plane $\mathcal{T}$. A **pentomino** is a connected patch made up of 5 squares, so that the patch is itself a tile (that is, the union of the 5 squares is a topological disk).

(a) There are 12 distinct pentominoes that, when viewed as a tile, admit a planar tiling. Showing all your work, find each pentomino. How do you know your list is complete, that is, can you conclude from your work that there are exactly 12 distinct pentominoes?

(b) Show that one of your pentominoes admits two distinct tilings. Justify your answer.

4. Consider the prototile T shown below together with monohedral tiling $\mathcal{T}$ studied in Lectures 7 and 8:



(a)
(b)

(a) Enumerate, with proof, all of the patches that can occur in a tiling of the plane by T consisting of three copies of T (write T_1, T_2, T_3) so that $T_1 \cap T_2 \cap T_3$ is a corner of each tile and $T_i \cap T_j$ is a complete side of T_i or T_j (possibly both) for all $1 \leq i, j \leq 3$. (Important: the condition that your patches arise as part of a tiling should shorten your list.)

(b) Enumerate, with proof, all of the patches consisting of four copies of T (write T_1, T_2, T_3, T_4) that are consistent with the constraints imposed by part (a) so that $T_i \cap T_j$ is empty or is a complete side of T_i or T_j (possibly both) and $T_i \cap T_j \cap T_k$ is empty for all $1 \leq i, j, k \leq 4$. (Important: without this consistency condition, your list will be much longer and not all patches will be relevant to a tiling with this prototile.)

Note: Part (a) and (b) refer to T only, and do not involve $\mathcal{T}$ per se.

(c) Prove that $\mathcal{T}$ is the unique tiling of the plane (up to equality of tilings) associated with T.