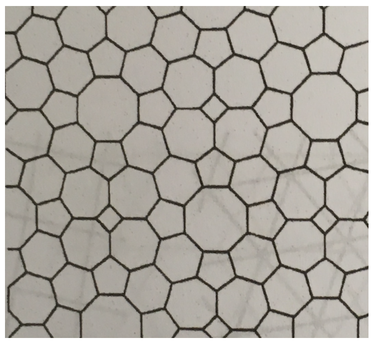
Math 308: Introduction to tilings

Assignment 3, due October 27 by 11:59 pm.

1. Give a detailed proof of the following assertion: The vertex type 4.5.20 does not arise in a uniform edge-to-edge tiling by regular polygons.

2. Figure 2.1.6 in Grünbaum and Shephard suggests the following edge-to-edge tiling by polygons:



Assuming such a tiling exists, prove that none of the polygons appearing is regular. Show your work.

In solving this problem, you can use the following simplifying assumptions:
(i) The 4-gon present may be a square; show that the remaining polygons cannot be regular.
(ii) Assume that there are 5 prototiles for this tiling

3. Consider the tilings (6^3) (also known as the honeycomb tiling) and the tilings $(3^6;3^4.6)_2$ and $(3^6;3^2.6^2)$ (compare Figure 2.2.1 on page 66 of Grünbaum and Shephard). Providing all details,

(a) prove that (6^3) is uniform;

(b) prove that $(3^6;3^4.6)_2$ and $(3^6;3^2.6^2)$ are distinct tilings; and

(c) prove that $(3^6;3^4.6)_2$ and $(3^6;3^2.6^2)$ are 2-uniform tilings.

4. Let a, b, c, d be integers greater than 2, and consider a regular a-gon, a regular b-gon, a regular c-gon, and a regular d-gon, all meeting at a single vertex. All polygons as convex; assume throughout that $a \leq b \leq c \leq d$.

(a) Show that

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} = 1$$

is a necessary condition for these 4 polygons to arise in an edge-to-edge tiling.

(b) Prove that $a \leq 4$.

(c) What is the unique solution $\{a, b, c, d\}$ when $a = 4$? Justify your answer.

(d) When $a = 3$, justifying your work, give a complete list of the possible quadruples $\{a, b, c, d\}$ giving rise to vertex species. **Hint:** It may be helpful to break into sub-cases conditioned on the value of b.