Math 308: Introduction to tilings

Assignment 4, due Friday November 10 by 11:59 pm.

1. Let $\mathcal{T}$ be a tiling by polygons, and let T be a tile of $\mathcal{T}$. Prove that if T has a corner that is not a vertex in $\mathcal{T}$ then $\mathcal{T}$ is not a proper tiling.

2. Consider a 4-gon with angles $\{\alpha, \beta, \frac{\pi}{2}, \frac{\pi}{2}\}$ such that $\alpha + \beta = \pi$ and $\alpha \neq \beta$; this is the tile T. Showing all your work, construct a tiling of type $\mathbf{P_4}$–$\mathbf{N}$ for some $\mathbf{N}$ having prototile T.

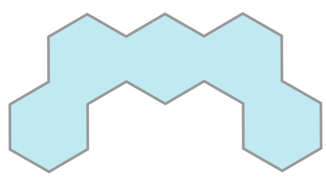
Note: Your solution can fix particular values of α and β if you find that helpful.

3. Showing all your work, prove that any two of the following four polygonal isohedral types of tilings by triangles are distinct: $\mathbf{P_3}$–$\mathbf{10}$, $\mathbf{P_3}$–$\mathbf{11}$, $\mathbf{P_3}$–$\mathbf{12}$, $\mathbf{P_3}$–$\mathbf{13}$

4. Given a tiling $\mathcal{T}$ write $T \asymp T'$ for tiles $T, T' \in \mathcal{T}$ whenever $T' = tT$ for some translation $t \in S(\mathcal{T})$.

(a) Verify that $\asymp$ is an equivalence relation on tiles.

The equivalence classes under $\asymp$ are called the *aspects* of a tile. Consider the 5-hexe prototile T shown (assume each of the 5 hexagons constructing this prototile are congruent and regular):



(b) Show that T admits no isohedral tiling with n aspects, where $n = 1, 2, 3$.

(c) Determine all isohedral tilings with with prototile T.