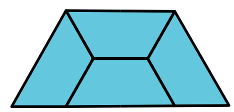
Math 308: Introduction to tilings

Assignment 5, due November 24 by 11:59 pm.

1. By giving an explicit planar similarity, prove that the Laves tiling $[4.8^2]$ (see Page 96 of Grünbaum and Shephard) is a similarity tiling.

2. Consider the following patch:



Use the decomposition of the trapezoid (into 4 smaller similar trapezoids) to generate a monohedral non-periodic 4-similarity tiling. Be sure to justify all of your work.

3. This problem shows that certain rotational symmetries rule out periodicity.

(i) Show that a rotation in the plane by an angle θ can be represented by a matrix with trace $2\cos\theta$.

(ii) Prove that $\mathrm{tr}(AB) = \mathrm{tr}(BA)$ when A and B are 2×2 matrices.

(iii) By appealing to part **(ii)**, show that regardless of what basis we use to express the rotation in part **(i)** the trace is unchanged.

(iv) Assume that a tiling $\mathcal{T}$ has a rotational symmetry *and* a translational symmetry. Prove that the trace of the rotation must be an integer.

(v) Deduce that if $\mathcal{T}$ has 5-fold rotational symmetry then it cannot admit a translational symmetry.

4. This problem considers the Amman prototiles described on Page 529 of Grünbaum and Shephard. You should assume throughout, where needed, that the double arrows run vertically and the triple arrows run horizontally.

(i) Show that, if the set of Amman prototiles admits a periodic tiling $\mathcal{T}$, then translations parallel to the sides of the modified squares must be included in the symmetry group $S(\mathcal{T})$.

(ii) Describe how to construct all possible faults that can occur in an Amman tiling.

(iii) Assuming that, away from these faults, the cornered tiles appear in alternating rows and columns, provide (with justification) a complete list of 3-by-3 blocks of Amman tiles that can occur away from the faults.

(iv) Prove that the Amman tiles are an aperiodic set.