Math 308: Introduction to tilings
Assignment 6, due December 7 by 11:59 pm.
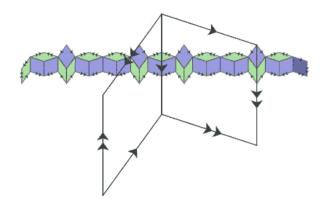
1.

(a) Let $\{T_1, T_2\}$ be a tile set admitting a periodic dihedral tiling $\mathcal{T}$. Prove that the relative density of T_1 tiles to T_2 tiles occurring in $\mathcal{T}$ must be a rational number.

Now consider the kite-dart tiling considered in class—if such a tiling exists, part (a) shows that it cannot be periodic. In class we established this by considering recombinations and counting kites (k_n) and darts (d_n) at each step.

(b) Calculate each of k_4 and d_4 in terms of k_1 and d_1.

(c) Find patches exhibiting both of the 4-fold compositions from part (b).

2.

(a) Given a Penrose tiling, prove that any given finite patch appears infinitely often in the tiling.

(b) Consider the following figure from Austin's notes:

Prove that the larger rhombs shown can be found in the fourth inflated tiling of a patch that contains the given strip (of smaller rhombs).