Math 309: Introduction to knot theory

Assignment 1, due Friday September 20 by 10:59 pm.

1. Recall, from class, that for a pair of relatively prime positive integers p and q we can form a knot with parametrization

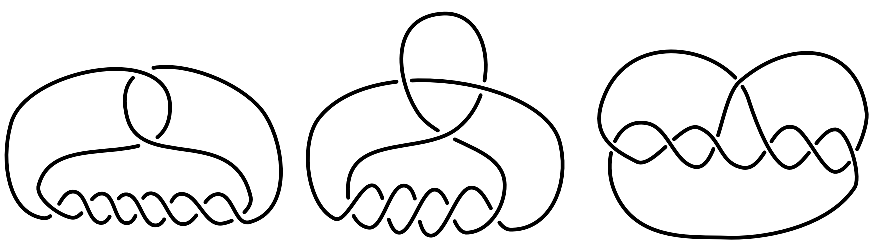
$$x(t) = (2 + \cos(pt))\cos(qt)$$
$$y(t) = (2 + \cos(pt))\sin(qt)$$
$$z(t) = \sin(pt)$$

where $0 \leq t \leq 2\pi$. This yields the (q, p)-torus knot, which is denoted $T(q, p)$.

(a) In the case $p = 1, q = 4$, consider projections to both the xy-plane and to the yz-plane. (Note that, just as in class, you should provide and make reference to graphs of the relevant coordinate functions.) In each case, either convert the projection into a knot diagram, or give a justification for why this is not possible.

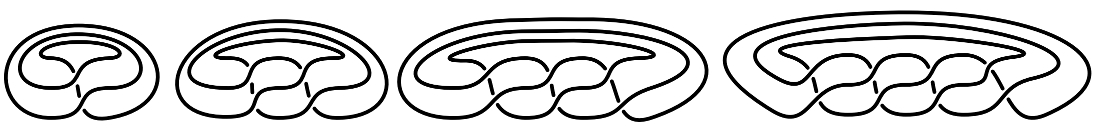
(b) Is the knot $T(4, 1)$ from part **(a)** trivial? Be sure to provide a full justification of your answer.

2. Decide the 3-colourability of each of the following knots:



In each case, either give a valid 3-colouring or prove that no 3-colouring exists.

3. Consider the torus links $T(3, p)$, for all integers $p > 0$. The following are the first 4 links of this infinite family:



(Reading from left to right, these are the cases $p = 1, 2, 3, 4$.) Determine, with proof, precisely which values of p yields a 3-colourable link.