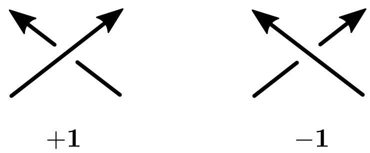
Math 309: Introduction to knot theory

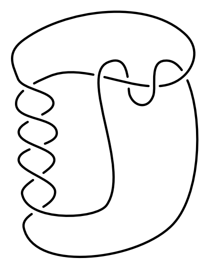
Assignment 2, due October 4 by 10:59 pm.

1. Fix a knot diagram D and make a choice of orientation on this diagram. Consider the following assignment at each crossing:



Define $\#(D)$ to be the sum over *all* crossings in D of values according to this assignment. Prove that $\#(D)$ is *not* a knot invariant.

2. Recall that McCoy's theorem states that an alternating knot with unknotting number 1 admits an unknotting crossing in any alternating diagram. Consider the following diagram:



For this problem, let this represent a knot referred to as K. You may assume that K is non-trivial.

(a) Show that, based on the diagram given, the unknotting number of K is *at most* 2. You should justify, using available invariants introduced so far in the course, any claims of non-triviality in your argument.

(b) Find an alternating diagram for K, and using this give a proof that the unknotting number of K is *exactly* 2.

3. A **simple link** is a 2-component link with the property that it admits a diagram that has *exactly* two crossings between the link components.

(a) Give an example, with proof appealing to the definition given, of a simple link.

(b) Give an example, again with proof, of a 2-component link that is *not* simple.

(c) Consider a simple link L with the property that each component is a 3-colourable knot. Prove that the link L is 3-colourable.