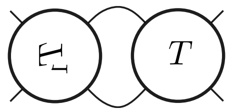
Math 309: Introduction to knot theory

Assignment 3, due Friday October 18 by 10:59 pm.

1. Recall that, given tangles T and T_1, we defined $T_1 * T$ as



and regarded this as "multiplication" for tangles (note that Adams uses the symbol $\cdot$ in place of $*$).

(a) Is there a tangle A such that $T' * A \sim T'$ for all tangles T'? Justify your answer.

(b) Is there a tangle B such that $B * T' \sim T'$ for all tangles T'? Justify your answer.

2. In this problem we will use the Conway notation for a rational tangle $[a_1, \ldots, a_n]$ to also denote the continued fraction

$$a_n + \cfrac{1}{a_{n-1} + \cfrac{1}{\ddots + \frac{1}{a_1}}}$$

(a) Express $\frac{25}{11}$ as a continued fraction $[a_1, \ldots, a_n]$ such that $a_n < 25/11$. With this constraint on a_n, is it possible to make it so that every a_i in the continued fraction is a positive integer? Justify your answer.

(b) Express $\frac{25}{11}$ as a continued fraction $[a_1, \ldots, a_n]$ such that $a_n > 25/11$. With this constraint on a_n, is it possible to make it so that every a_i in the continued fraction is a positive integer? Justify your answer.

(c) Draw the rational tangle associated with each of the continued fractions you found in parts **(a)** and **(b)**. Is either tangle alternating?

(d) By manipulating the tangle diagrams, show that the tangles from part **(c)** are equivalent. (You do not need to list every Reidemeister move, just show your steps clearly.)

3. Let $0 < p < q$ where p is an even integer and q is an odd integer. Prove that $L(T)$ is a 2-component link when T is the rational tangle associated with the fraction $\frac{p}{q}$.