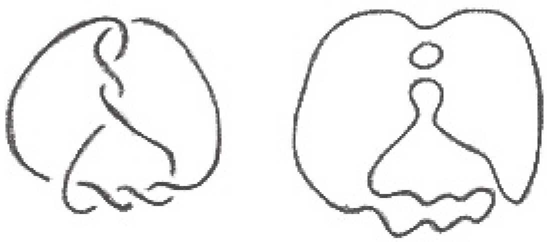
Math 309: Introduction to knot theory

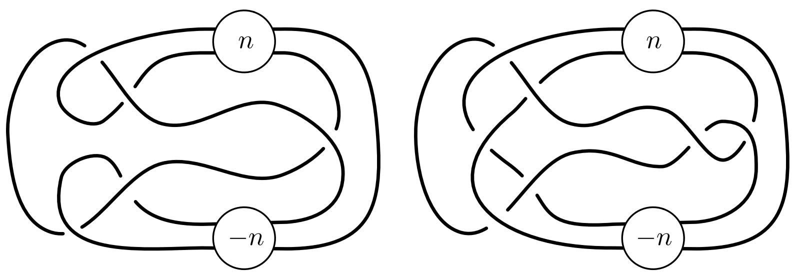
Assignment 5, due Friday November 15 by 10:59 pm.

1. Consider the following diagram (left) with associated state S_1 (right):



List all states S_2 that can be obtained from S_1 by switching one resolution so that $b(S_2) = b(S_1)+1$. Be sure to justify why your list of states is complete.

2. Prove that the following knot diagrams have the same Jones polynomial for all integers $n > 0$,



where the tangles indicated are rational tangles associated with the relevant integers.

3. Suppose D is a knot diagram that is both alternating and reduced.

(a) Let S_A be the state obtained from D by choosing the A-split at every crossing. Show that the highest power of A contributed to $\langle D \rangle$ by S_A is strictly larger than that of any state for D with exactly one B-split.

(b) Let S_B be the state associated with D that is obtained by considering B-splits are every crossing. Show that the bottom power that this state contributes to the bracket polynomial is $-n-2(\mathcal{S}-1)$, where n is the number of crossings in D and $\mathcal{S}$ is the number of shaded regions in the checkerboard shading of D.