TOPICS IN METRIC GEOMETRY AND GEOMETRIC GROUP THEORY (SYLLABUS FOR MATH 602D)

LIOR SILBERMAN

Course Website	`http://www.math.ubc.ca/~lior/teaching/602D_F08/`
My Website	`http://www.math.ubc.ca/~lior/`
My Office	MAT 229B — 604-827-3031
Class	MWF 10-11am at MATX 1102
Office Hours	By appointment; regular hours TBA

I shall discuss some examples of the interaction between group theory and the geometry of metric spaces, leading toward Gromov's theorem classfiying groups of polynomial growth. This course lies at the intersection between algebra, combinatorics and analysis. We will see how we can understand groups using their actions on metric spaces, and also how to understand the algebraic structure of a group by considering the group itself as a metric space. Along the way we will learn ideas in graph theory and representation theory. Time permitting we will continue to current research in the field.

Prerequisites. Undergraduate analysis in $\mathbb{R}^n$ (completeness, compactness). Introductory group theory.

What the course will include (rough outline).

(1) Basic constructions in metric spaces.
- Examples of metric spaces (Euclidean space, hyperbolic space, trees)
- Basic constructions
- Limits of metric spaces

(2) Some group theory
- Solvable and nilpotent groups.
- Free groups; generators and relations.
- Topological groups and group actions.

(3) Averaging and fixed-point properties: property (T) and expander graphs.
- Fixed point of involutions.
- Averaging on graphs; expanders.
- Averaging on groups and property (T).

(4) Gromov's theorem on groups of polynomial growth (see reverse).

Administrivia. There will be regular homework in the first half of the course.

A good reference for theory of metric spaces is the book [1]. For Gromov's theorem we shall use the original paper [2] and the recent preprint [3].

Date: September 2, 2008.

GROMOV'S THEOREM

Let Γ be a group generated by the finite symmetric set S. For each integer r let $B(r) \subset \Gamma$ be those elements than can be written as the product of at most r elements of S.

Theorem. *The sequence $\#B(r)$ grows polynomially in r (that is, $\#B(r) \leq Cr^d$ for some $C, d > 0$) if any only if Γ has a nilpotent subgroup Γ' of finite index.*

REFERENCES

[1] Martin R. Bridson and André Haefliger. *Metric spaces of non-positive curvature*, volume 319 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, Berlin, 1999.

[2] Mikhael Gromov. Groups of polynomial growth and expanding maps. *Inst. Hautes Études Sci. Publ. Math.*, (53):53–73, 1981.

[3] Bruce Kleiner. A new proof of gromov's theorem on groups of polynomial growth. arXiv:math.GR/0710.4593, 2007.