

Math 100 §105, Fall Term 2010
Sample Midterm Exam

November 8th, 2010

Student number:

LAST name:

First name:

Instructions

- Do not turn this page over until instructed. You will have 45 minutes for the exam.
- You may not use books, notes or electronic devices of any kind.
- Solutions should be written clearly, in complete English sentences, showing all your work.
- If you are using a result from the textbook, the lectures or the problem sets, state it properly.

Signature:

1		/18
2		/8
3		/4
4		/10
Total		/40

1 Short-form answers

Show your work and clearly delineate your final answer. Not all problems are of equal difficulty.

[3] a. If $x^2y^2 + x \sin y = 4$, find $\frac{dy}{dx}$.

[3] b. Let $f(x) = x^3 \ln x$. Find the $f^{(4)}(x)$, the fourth derivative of f .

[3] c. Differentiate $(\tan x)^x$.

[3] d. Write down the first three nonzero terms in the Maclaurin series for $x \sin(-2x)$.

[3] e. Use a linear approximation to approximate $\sqrt{100.2}$.

[3] f. Give an upper bound for the error in your answer to part e.

2 Long-form answers

The normal temperature of your Vancouver apartment is 23 degrees; the daytime temperature outside is about 5 degrees.

[4] A window remains open when you leave for UBC at 7am. By 1pm, the temperature in the apartment has dropped to 11 degrees. What will the temperature be at 7pm?

3 Long-form answers

[8] A trough is 10 m long and its ends have the shape of equilateral triangles (i.e. all three sides have equal length) that are 2 m across, situated with their points down. If the trough is being filled with water at the rate of $12\text{m}^3/\text{min}$, how fast is the water level rising when the water is 60cm deep?

4 Long-form answers

Consider the function $f(x) = \sqrt{1 - xe^{-x/a}}$ on the interval $[0, 1]$. Here a is a positive parameter. Do one of parts (a), (a')

[5] a. Find the absolute maximum of f on the interval.

[5] a'. Find the absolute minimum of f on the interval.

[2] b. Let $F(a)$ be your answer to part a / a'. Assuming that a is very small, write down a linear approximation to $F(a)$.

[3] b. Find the absolute minimum and maximum of $f(x) = e^{-|x|}$ on the interval $[-10, 10]$. Where are they attained?