

Math 100, Section 105
Problem Set 2

Due: October 25th, 2010

Student number:

LAST name:

First name:

Mark:

Instructions

- Please print this sheet out and write your student number, last name (all capitals) and first name. Please use your “official” name as it appears in the student registry even if you prefer to be called by another name — this is needed for the grader to enter your grade in the system.
- Use this page as a cover sheet for your solutions, but do not write your solutions on it. STAPLE your pages together; lost pages are your responsibility.
- Due at the beginning of class on the date indicated; late work will not be accepted.
- Place in the pigeonhole corresponding to the first letter of your last name.

Problems

1. Differentiate the following functions: (a) $(t^3 + 2t)e^t$ (b) $\frac{\sin x}{x^4}$ (c) $e^{\cos x}$ (d) $\arcsin(x^3)$ (e) $x^{\sin x}$ (f) $\frac{x}{3+e^x}$ (g) $t^3 \cos t$.
2. Find $\frac{dy}{dx}$ in terms of x and y if: (a) $x^2 + xy - y^2 = 4$ (b) $x^2 = y^3 + ay + b$.
3. (Final exam 2009) Find all values of the constant c that makes the following function continuous everywhere, or show that such values do not exist:

$$f(x) = \begin{cases} \frac{\sin(4x)}{x} & x \neq 0 \\ c & x = 0 \end{cases}.$$

(continue over)

4. The motion of an electron with energy E is moving along a 1-dimensional wire is described by a *wavefunction* $f(x)$. There is an impurity in the wire, between $-L \leq x \leq L$, so that the function f satisfies the following equations:

$$f''(x) = \begin{cases} -Ef(x) & |x| < L \\ (V - E)f(x) & |x| > L \end{cases}.$$

- (a) Assume $0 < E < V$. Show that the following is a solution to the equation for $|x| \neq L$.

$$f(x) = \begin{cases} \cos(\sqrt{E}x) & |x| < L \\ Ae^{-\sqrt{V-E}|x|} & |x| > L \end{cases}.$$

- (b) For f to actually describe the motion of a particle, both f and f' must be continuous at $x = L$. Derive two independent equations that E and A must satisfy for this to be true.
- (c) Show that the continuity conditions require the energy to satisfy $\tan(\sqrt{E} \cdot L) = \sqrt{\frac{V-E}{E}}$. In particular, not every E can be the energy of the electron (this phenomenon is known as “quantization” of the energy).
- (d) Find $\frac{dE}{dV}$ in terms of E, V (and the fixed length L). What happens to the energy level E as the effect of the impurity increases?
5. Repeat the previous problem parts (a)-(d) but using the solution

$$f(x) = \begin{cases} \sin(\sqrt{E}x) & |x| < L \\ B \operatorname{sgn}(x)e^{-\sqrt{V-E}|x|} & |x| > L \end{cases}$$

instead. Here $\operatorname{sgn}(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$. The equation you derive in part (c) will be slightly different, and this will effect part (d).