Math 100, Section 105
Problem Set 3

Due: November 3rd, 2010

Student number:

LAST name:

First name:

Mark:

Instructions

- Please print this sheet out and write your student number, last name (all capitals) and first name. Please use your "official" name as it appears in the student registry even if you prefer to be called by another name — this is needed for the grader to enter your grade in the system.
- Use this page as a cover sheet for your solutions, but do not write your solutions on it. STAPLE your pages together; lost pages are your responsibility.
- Due at the beginning of class on the date indicated; late work will not be accepted.
- Place in the pigeonhole corresponding to the first letter of your last name.

Problems

1. Find Taylor polynomials of the required degree for the following functions about the given points. (a) $f(x) = \cos(6x)$, degree 4 about $x = 0$; (b) $f(x) = \sin(6x)$, degree 4 about $x = \frac{\pi}{24}$; (c) $f(x) = 5x^2+3x+2$, degree 2 about $x = 0$; (d) $f(x) = 5x^2 + 3x + 2$, degree 2 about $x = 1$; (e) $f(x) = e^{x^2}$, degree 7 about $x = 0$; (f) $f(x) = \sqrt{1 + x^3}$, degree 2 about $x = 2$.

2. (Final exam 2005)

   (a) Suppose we know that $f(1) = f'(1) = 1$. Define $g(x) = f(x^3)$. Use a linear approximation to the function g (not a linear approximation to the function f) to estimate $g(1.1)$.

   (b) Use a suitable linear approximation to estimate $(17)^{1/4}$. Give your answer as a fraction with integer numerator and denominator.
   Hint: (**not given on the exam**) $16^{1/4} = 2$.

3. Find the constant. linear, quadratic and cubic Taylor polynomials for $\arcsin(x)$ near $x = 0$ and use them to make three approximations to $6\arcsin(\frac{1}{2})$. What is the correct value?

(continue over)

4. (Final exam 2005) "A circular Ferris wheel with radius 10 metres is revolving at the rate of 10 radians per minute. How fast is a passenger on the wheel rising when the passenger is 6 metres higher than the centre of the wheel and is rising? Include units in your answer."

   (a) Let θ be the angle between the horizontal and the line connecting the center of the wheel and the passenger. What is $\frac{d\theta}{dt}$?

   (b) Write an equation relating the height of the passenger to θ.

   (c) How fast is a passenger on the wheel rising when the passenger is 6 metres higher than the centre of the wheel and is rising? Include units in your answer.

   REMARK: when this was given in the exam, the problem consisted of just the quote above. In other words, from now on you are supposed to come up with intermediate steps like (a),(b) on you own without prompting.

5. At noon, a sailboat is 20km due south of a freighter. The sailboat is traveling due east; t hours after noon its distance from its initial point is $x(t)$ kilometers. The the freighter is traveling due south; t hours after noon its distance from its initial point is $y(t)$ kilometers. *Assume the Earth is flat.*

   (a) What is the distance between the freighter and the sailboat, t hours after noon? (your answer will depend on the unknown functions x and y).

   (b) How fast is the distance changing? Your answer should depend on $x(t), y(t)$ and on the instantaneous velocities at time t.

   (c) Assume that the sailboat is travelling at constant speed of 20km/hr, the freighter at a constant speed of 40km/hr. What are $x(t)$, $y(t)$?

   (d) What is the rate of change of the distance, purely as a function of t?