Math 100, Section 105
Problem Set 3

Due: November 17th, 2010

Student number:
LAST name:
First name:
Mark:

Instructions

- Please print this sheet out and write your student number, last name (all capitals) and first name. Please use your "official" name as it appears in the student registry even if you prefer to be called by another name — this is needed for the grader to enter your grade in the system.
- Use this page as a cover sheet for your solutions, but do not write your solutions on it. STAPLE your pages together; lost pages are your responsibility.
- Due at the beginning of class on the date indicated; late work will not be accepted.
- Place in the pigeonhole corresponding to the first letter of your last name.

Problems

1. Differentiate the following functions (x is the variable, all other letters are constants): (a) $\sqrt{x^2+1}\sin x$ (b) $\cos(e^x)$ (c) $\arcsin(x^x)$ (d) $\frac{1}{1+x}$ (e) $\ln\left(2\frac{x+1}{x+2}\right)$ (f) $e^{ax}\sin(bx)$.
2. Find $\frac{dy}{dx}$ given the following equations: (a) $x+y=1$ (b) $\cos y + axy = 2x^2$ (c) $\sin 2y = 2\sin x\cos y$.
3. (Horizontal asymptotes) We say f has a *horizontal asymptote* at $+\infty$ if $\lim_{x\to+\infty} f(x)$ exists (similarly at $-\infty$). Which of the following functions have a horizontal asymptote at $+\infty$? At $-\infty$? at both? (a) $\frac{1}{1+x}$ (b) $\sin x$ (c) e^x (*d) $\frac{e^x-e^{-x}}{e^x+e^{-x}}$ (e) $\ln(1+x)-\ln x$ (f) x^2-2x.
4. For each function (i) find its domain (ii) divide the domain into open intervals on which f', f'' are non-zero; list the sign of f', f'' on each such interval (iii) for the endpoints of the intervals (where $f'=0$ or $f''=0$) say whether the point is an inflection point, critical point, local maximum, or local minimum.
(a) x^2+1 (b) $\frac{x}{x^2-1}$ (c) $\frac{x}{x^2+1}$ (d) $\frac{e^x}{e^{2x}-1}$.

(example on the reverse)

- Example: $f(x) = x^4 + 4x^3 - 8x^2$ is defined everywhere. It has a local maximum at $x = 0$ and local minima at $x = -4, 1$

x	$(-\infty, -4)$	-4	$\left(-4, -1-\sqrt{\frac{7}{3}}\right)$	$-1-\sqrt{\frac{7}{3}}$	$\left(-1-\sqrt{\frac{7}{3}}, 0\right)$	0
f'	-	0	+	+	+	0
f''	+	+	+	0	-	-

x	0	$\left(0, -1+\sqrt{\frac{7}{3}}\right)$	$-1+\sqrt{\frac{7}{3}}$	$\left(-1+\sqrt{\frac{7}{3}}, 1\right)$	1	$(1, \infty)$
f'	0	-	-	-	0	+
f''	-	-	0	+	+	+

5. Consider the curve $ay^2 = (a^2+1)x^3 + ax - (a+1)$, with a a positive parameter.
   (a) Show that the point $P = (1, \sqrt{a})$ lies on the curve and find the slope of the curve at P.
   (b) Find the positive value of a that maximizes that slope, or show that no such value exists.
   (c) Find the positive value of a which minimizes that slope, or show that no such value exists.

6. (The naive Laffer curve) Assume that at price p dollars, the widget industry will produce $S(p) = 10^5 p$ widgets, while consumers will be willing to buy $D(p) = 10^6\left(1 - \frac{p}{10}\right)$ widgets (if $p > 10$ no one will want to buy any widgets).
   (a) The market price p_M is determined by the equation $S(p_M) = D(p_M)$ ("supply = demand"). What is p_M? What is the quantity sold (that would be $S(p_M)$)

   Assume now that the government puts in a sales tax at the rate r so if the item is priced at p the buyers have to pay a total of $(1+r)p$ of which the industry only gets p and the government gets rp. This means that at price p manufactures will still offer $S(p)$ widgets but consumers will only demand $D((1+r)p)$ widgets.
   (b) Find the new market price $p_M(r)$ by solving the equation $S(p_M(r)) = D((1+r)p_M(r))$.
   (c) Total government revenue $R(r) = (rp_M(r)) \cdot (S(p_M(r)))$ is given by the product of the tax collected per unit $(r \cdot p_M(r))$ and the number of units sold $(S(p_M(r))$. Find the value of $0 \le r$ that will maxize government revenue.

   Remark: The curve $R(r)$ is known as the *Laffer curve*.