Math 121: Problem set 11 (due 4/4/12)

Practice problems (not for submission!)

Sections 9.5-9.7

Sequences and Series

1. Find the center, radius, and interval of convergence for the following series:
   (a) $\sum_{n=0}^{\infty} \frac{10e^n+5}{n^n} x^n$
   (b) $\sum_{n=0}^{\infty} \frac{\sqrt{n+5}}{4^n} (3x-1)^n$

2. Evaluate the following sums:
   (a) $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n3^n}$.
   (b) $\sum_{n=0}^{\infty} \frac{1}{n^2 2^n}$ (express your answer as an integral).
   (*c) $x^3 - \frac{x^{13}}{3! \cdot 9} + \frac{x^{23}}{5! \cdot 81} - \frac{x^{33}}{7! \cdot 729} + \cdots$
   (*d) $1 + \frac{x^2}{3!} + \frac{x^4}{5!} + \cdots = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} x^{2n}$.
   PRAC $\sum_{n=0}^{\infty} (n^2 - 3n + 5) x^n$ where $|x| < 1$.

3. Expand
   (a) $\frac{1}{x^3}$ about $x_0 = 2$ (Hint)
   (b) $\frac{x}{1+3x^2}$ in powers of x.
   (c) $\int_1^{1+x} \frac{\log t}{t-1}\, dt$ in powers of x.
   PRAC $\frac{1}{x^3+x^2-5x+3}$ in powers of x.

4. (Taylor polynomials)
   (a) Evaluate $\lim_{x \to 0} \frac{\sin(\sin x) - x\cos\left(\frac{7}{3}x\right)}{\arctan x (e^x - 1 - x)^2}$
   (b) Find r so that $A = \lim_{x \to 0} x^r \frac{e^{\sin x} - e^x}{e^{\cos x - 1}}$ exists and is non-zero, and evaluate A.

5. Define a sequence by $a_0 = a_1 = 1$ and $a_{n+2} = \frac{2n-1}{(n+1)(n+2)} a_n$ if $n \geq 0$.
   (a) Show that the series $\sum_{n=0}^{\infty} a_n x^n$ converges for all x.
   (b) Let $f(x)$ denote the sum of the series in part (a). Show that $f''(x) - 2xf'(x) + f = 0$.

Hint for 2(c): Differentiate the unknown function.
Hint for 3(a): Expand $\frac{1}{x}$ first, then differentiate.
Hint for 3(d): Partial fractions.