

1. Evaluate the limit

$$\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x^2 - 9}$$

**Solution:**  $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x^2 - 9} = \lim_{x \rightarrow 3} \frac{(x-3)(x+4)}{(x-3)(x+3)} = \lim_{x \rightarrow 3} \frac{x+4}{x+3} = \frac{\lim_{x \rightarrow 3}(x+4)}{\lim_{x \rightarrow 3}(x+3)} = \frac{7}{6}$ .

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2. Determine the infinite limit

$$\lim_{x \rightarrow -3^+} \frac{x + 2}{x + 3}$$

**Solution:** For  $-2 > x > -3$  we have  $x + 3 > 0$  but  $x + 2 < 0$  so for  $x$  close to  $-3$  but above it  $\frac{x+2}{x+3} < 0$ . Also,  $\lim_{x \rightarrow -3}(x + 3) = 0$  and  $\lim_{x \rightarrow -3}(x + 2) = -1 \neq 0$ . It follows that  $\lim_{x \rightarrow -3^+} \frac{x+2}{x+3} = -\infty$ .

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3. Let  $f(x)$  be such that  $x \cos x \leq f(x) \leq x(1 + e^x)$  for all  $x \geq 0$ . Find  $\lim_{x \rightarrow 0^+} f(x)$ .

**Solution:**  $\lim_{x \rightarrow 0}(x \cos x) = 0 \cdot \cos 0 = 0$  and  $\lim_{x \rightarrow 0}(x(1 + e^x)) = 0(1 + e^0) = 0$ . By the squeeze theorem it follows that  $\lim_{x \rightarrow 0^+} f(x) = 0$  too.