1. Find $\frac{dy}{dx}$ by implicit differentiation if

$$e^{x/y} = x - y\,.$$

Solution: We differentiate both sides, and find:

$$1 - \frac{dy}{dx} = \frac{d}{dx}\left(e^{x/y}\right) \overset{\text{chain}}{=} e^{x/y}\frac{d}{dx}\left(\frac{x}{y}\right) \overset{\text{quot}}{=} e^{x/y}\left(\frac{1}{y} - \frac{x}{y^2}\frac{dy}{dx}\right)\,.$$

Rearranging this reads:

$$\left[\frac{x}{y^2}e^{x/y} - 1\right]\frac{dy}{dx} = \frac{1}{y}e^{x/y} - 1$$

that is

$$\frac{dy}{dx} = \frac{\frac{1}{y}e^{x/y} - 1}{\frac{x}{y^2}e^{x/y} - 1} = \frac{ye^{x/y} - y^2}{xe^{x/y} - y^2}\,.$$

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2. Differentiate

$$y = \sqrt{\arctan x}\,.$$

Solution: We can write this as $\arctan x = y^2$ and hence $\tan y^2 = x$. We now differentiate, using that $\frac{d(\tan y)}{dy} = 1 + \tan^2 y$ to get

$$2y(1 + \tan^2 y^2)\frac{dy}{dx} = \frac{dx}{dx} = 1$$

so

$$\frac{dy}{dx} = \frac{1}{2y(1 + \tan^2 y^2)} = \frac{1}{2\sqrt{\arctan x}(1 + x^2)}\,.$$

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3. Find y' at the point where $x = 0$ if

$$xy + e^y = e\,.$$

Solution: Setting $x = 0$ we find $e^{y(0)} = e = e^1$ so $y(0) = 1$ and the point on the curve is $(0, 1)$. We now differentiate both sides, using the product rule and the chain rule to get:

$$y + x\frac{dy}{dx} + e^y\frac{dy}{dx} = 0$$

so

$$\frac{dy}{dx} = -\frac{y}{x + e^y}\,.$$

When $x = 0$ and $y = 1$ this reads

$$y'(0) = -\frac{1}{e}\,.$$