MATH 100 – WORKSHEET 16
APPLICATIONS

(1) The position of a particle at time t is given by $f(t) = t\sin(\pi t)$.
   (a) Find the velocity at time t, and specifically at $t = 3$.
   $v(t) = f'(t) \overset{\text{pdt}}{=} \sin(\pi t) + \pi t\cos(\pi t)$. Now $\sin(3\pi) = \sin\pi = 0$ and $\cos(3\pi) = \cos\pi = -1$ so $v(3) = -3\pi$
   (b) When is the particle accelerating? Decelerating? At rest?
   $a(t) = v'(t) = 2\pi\cos(\pi t) - \pi^2 t\sin(\pi t)$.
   (c) Find the total distance travelled by $t = 5$. $S =$

(2) A ball is falling from rest in air. Its height at time t is given by

$$h(t) = H_0 - gt_0\left(t + t_0 e^{-t/t_0} - t_0\right)$$

where H_0 is the initial height and t_0 is a constant.
   (a) Find the velocity of the ball. $v(t) = h'(t) = 0 - gt_0\left(1 + t_0 e^{-t/t_0}\left(-\frac{1}{t_0}\right) - 0\right) = -gt_0\left(1 - e^{-t/t_0}\right)$.
   (b) Find the acceleration. $a(t) = v'(t) = -gt_9\left(0 - e^{-t/t_0}\left(-\frac{1}{t_0}\right)\right) = -ge^{-t/t_0}$.
   (c) Find $\lim_{t\to\infty} v(t) = \lim_{t\to\infty}\left(-gt_0\left(1 - e^{-t/t_0}\right)\right) = -gt_0\left(1 - \lim_{t\to\infty} e^{-t/t_0}\right) = -gt_0(1-0) = -gt_0$.

(3) Water is filling a cylindrical container of radius $r = 10$cm. Suppose that at time t the height of the water is $\left(t + t^2\right)$ cm. How fast is the volume growing?
Solution: At time t the water forms a cylinder of radius r and height $(t + t^2)$ hence volume $V(t) = \pi r^2(t + t^2)\text{cm}^3$. It follows that $V'(t) = \pi r^2(1 + 2t) = 100\pi(1 + 2t)$.

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(4) A spherical baloon is expanding, so that at the time its radius is 15cm, the radius is growing at the rate of 1cm/sec. How fast is the volume of the baloon growing?

Solution: $V(t) = \frac{4}{3}\pi\left(r(t)\right)^3$ so $V'(t) = \frac{4}{3}\pi \cdot 3(r(t))^2 r'(t)$. At the given time we have $r = 15$, $r' = 1$ so $V'(t) = 4\pi \cdot 15^2 \cdot 1 = 900\pi \frac{\text{cm}^3}{\text{sec}}$.

(5) A rocket is flying in space. The momentum of the rocket is given by the formula $p = mv$, where m is the mass and v is the velocity. At a time where the mass of the rocket is $m = 1000\text{kg}$ and its velocity is $v = 5000\frac{\text{m}}{\text{sec}}$ the rocket is accelerating at the rate $a = 20\frac{\text{m}}{\text{sec}^2}$ and losing mass at the rate $10\frac{\text{kg}}{\text{sec}}$. Find the rate of change of the momentum with time.

Solution: $\frac{dp}{dt} \overset{\text{pdt}}{=} \frac{dm}{dt}v + m\frac{dv}{dt} = (-10)5000 + 1000 \cdot 20 = 20,000 - 50,000 = -30,000\frac{\text{km m}}{\text{sec}}$.

(6) A metal rod of length 30cm has total mass 800gr. Suppose that the mass of the part of the rod between the left end and a point $x$cm to the right has mass $\left(x^2 - \frac{x^3}{270}\right)$ gr. Find the density of the rod in its middle.

Solution: The density is given by the derivative of the mass. Thus $d(x) = \left(2x - \frac{x^2}{90}\right)\frac{\text{gr}}{\text{cm}}$. At $x = 15$ this gives $d(15\text{cm}) = 30 - \frac{225}{90} = 30 - 2.5 = 27.5$.